Heuristics in the Evolution of Units of Account and Media of Exchange

Thomas Marmefelt

Associate Professor of Economics

University of Södertörn, Social Sciences, SE-141 89 Huddinge, Sweden

Phone: +46 8 608 41 15; E-mail: thomas.marmefelt@sh.se

Adjunct Professor (Docent) of Economics, especially Evolutionary Economics

Åbo Akademi University, Åbo/Turku, Finland

Paper prepared for the Third WINIR Conference: Institutions & Human Behavior

Boston, September 2-5, 2016

Preliminary version

August 2016

Heuristics in the Evolution of Units of Account and Media of Exchange

Thomas Marmefelt
University of Södertörn, Department of Social Sciences, SE-141 89 Huddinge, Sweden
Phone: +46 8 608 41 15, Fax: +46 8 608 40 30, E-mail: thomas.marmefelt@sh.se
Åbo Akademi University, Åbo/Turku, Finland

Abstract

The co-evolution of units of account and media of exchange with trade is of particular importance to money as emergent order. Money is a social institution, a clearing device, and monetary arrangements are complex evolving systems. The functioning of money as a social institution, involves some monetary heuristics that evolve over time, being fast and frugal. Complexity implies that agents are more or less ignorant, having very incomplete information, and try to develop simple heuristics or rules of thumb, such as imitation. Economic agents use a map, script, schema, or classifier system to reproduce their environment, including the monetary arrangement. Monetary heuristics evolve continually, when humans interact in the economic environment, in which they conduct exchange, what we call the market. The evolution of units of account and media of exchange are studied as adaptive responses by human minds. As social institution, money has both a cognitive dimension and a behavioural dimension. The cognitive dimension represents the way traders think about money as unit of account and medium of exchange, establishing monetary heuristics, which yields the perceived purchasing power of currencies. The behavioral dimension represents how agents act upon perceived purchasing power of money; what commodity bundle could be bought for a unit of a currency, a medium of account with its associated media of exchange, for another currency, thus establishing exchange rates. Hence, exchange rates between currencies are established according to relative perceived purchasing power, using heuristics which define a classifier system. Actual exchange rates reflect monetary heuristics of traders. Using stylized facts from four historical cases, this paper analyzes how to formalize monetary evolution as an ongoing loop between monetary heuristics and exchange rates.

Keeywords: heuristics, unit of account, medium of exchange

1 INTRODUCTION

The co-evolution of units of account and media of exchange with trade is of particular importance to money as emergent order. As Clower (1999) argues, monetary exchange arises from the same forces of self-interest that induce individuals to make markets what constitutes money depends on when and where we are there, i.e. on historical context. Pre-modern Europe, was characterized by a separation of unit of account from medium of exchange, represented by two separate monies:

the money of account and the real money (see e.g. Einaudi, 1936; Einzig, 1966). Eianaudi calls the money of account imaginary money, which Roover (1948) rejects, finding the money of account in medieval Flanders based upon either a real coin or a coin that had ceased to circulate but still represented a weight in gold or silver. Similarly, Honda (2007) finds in fifteenth century Japan, the emergence of a division between "pure coins" used as measure of value and "ordinary coins" used as means of payment. While Menger (1892a,b) argues that money evolved as a generally acceptable media of exchange, being the most salable or liquid of assets, Cowen and Krozner (1994) argue that the development of media of account is historically and logically prior to the development of media of exchange. The medium of account and medium of exchange functions were eventually unified into a single asset, money, with the establishment of the gold standard.

Money is a social institution, representing some purchasing power, allowing for the settlement of accounts, following Wicksell ([1906] 1966), Mises ([1912] 1924), and Schumpeter (1917-18, 1970). Menger (1892a,b) considers money as a social institution, because he stresses the formation of expectations concerning the acceptability of money and that learning of the acceptability as the driving force behind the emergence of money. As social institution, money has a cognitive dimension, which represents the way traders think about money as unit of account and medium of exchange, respectively, in the form of monetary heuristics, but money also has a behavioral dimension, which is expressed in the purchasing power of money. The underlying view is that of Aoki (2011), according to whom in the cognitive dimension of institutions, behavioral beliefs are inferred from a public representation of the state of play in the cognitive dimension; in the behavioral dimension of institutions, strategic choices motivated by behavioral beliefs generates a state of play. The functioning of money as a social institution, involves some monetary heuristics that evolve over time, being fast and frugal. Complexity implies that agents

are more or less ignorant, having very incomplete information, and try to develop simple heuristics or rules of thumb, such as imitation. Economic agents use a map, script, schema, or classifier system to reproduce their environment, including the monetary arrangement. Monetary heuristics evolve continually, when humans interact in the economic environment, in which they conduct exchange, what we call the market. The evolution of units of account and media of exchange are to be regarded as adaptive responses by human minds.

Gigerenzer (2008) defines fast and frugal heuristics, as a strategy that searches for minimal information and consists of building blocks that exploit evolved capacities and environmental structures, while social heuristics exploit the capacity of humans for social learning and imitation. Heuristics are devices to address complexity, because complexity implies that agents are more or less ignorant, having very incomplete information, and therefore try to develop simple heuristics or rules of thumb, such as imitation.

Economic agents use a metal model Hayek (1952) considers the map, which reproduces relations in the physical world, around us. Schank and Abelson (1977) use the notion of a script, which links events to subsequent outcomes, in which common knowledge of the script implies that everyone knows the meaning the actions of other agents and how to respond. Turning to complex adaptive systems, Gell-Mann (1994) calls it a schema, which represents perceived regularities, being some combination of description, prediction, and prescriptions for action, while Holland (2006) uses the notion of classifier systems to define a classifier system agent, composed of lists of classifiers and signals and sets of detectors, effectors (filling needs), and reservoirs (the agent's needs).

Monetary heuristics evolve continually, when humans interact in the economic environment, in which they conduct exchange, what we call the market. The evolution of units of account and media of exchange are studied as adaptive responses by human minds. The behavioral dimension

represents how agents act upon perceived purchasing power of money; what commodity bundle could be bought for a unit of a currency, a medium of account with its associated media of exchange, for another currency, thus establishing exchange rates. Hence, exchange rates between currencies are established according to relative perceived purchasing power, using heuristics which define a classifier system. It is therefore plausible to assume that actual exchange rates reflect monetary heuristics of traders. Using stylized facts from four historical cases, this paper analyzes how to formalize monetary evolution as an ongoing loop between monetary heuristics and exchange rates.

2 STYLIZED FACTS BASED UPON FOUR HISTORICAL MONETARY REGIMES

The four historical cases are four historical monetary regimes in the Baltic and North Seas Region, a region starting in the periphery of long-distance trade, then becoming the core, before becoming more peripheral again. The four regimes considered are: Hanseatic Monetary Arrangements; Seventeenth Century Exchange Banks; the Gold Standard and Monetary Unions; and Monetary Fragmentation and the Gold-Exchange Standard, the financial center shifting from Bruges, connected to Venice and Genoa, Amsterdam, London, and finally London, dependent on New York.

Starting with Hanseatic Monetary Arrangements, we focus on the Baltic and North Seas region. This region formed a northern counterpart to the Mediterranean (Dollinger, 1998). The Hanseatic League, an association of merchants and later of cities, played a leading role in the trading area that emerged from the Low Countries to the Baltic, along a trading axis from Novgorod in the east to London in the west, including Lübeck, Hamburg, and Bruges (Fink, 2011; Kohn 2001a,b; Põder, 2010). Nevertheless, German influence was limited up to the twelfth century, but the Hanseatic League evolved as cities emerged, starting with the foundation of

Lübeck in 1158-59 with the aim to promote long-distance trade, favoring Baltic trade, establishing trade relations between Lübeck and Gotland, Russia, and Sweden, while there was an old trade link in the North Sea between Bremen and Bergen in Norway (Dollinger, 1998). Hence, there were some north-south axes connecting to the east-west axis. Bruges was the focal point, where merchants from Lübeck, Visby, Rostock, Elbing, and Riga, the so called *Osterlinge*, connected to the world market of the West, indeed the meeting place of Italian and Hanseatic merchants and the point of contact between Mediterranean and Baltic commerce (Roover, 1948; Dollinger, 1998).

Bruges also played a central role in monetary exchange. Italian merchant-bankers, involved in bills of exchange business, established in the thirteenth century branches in Bruges, where Hanseatic merchants, who lacked an exchange market of their own, used agents (Roover, 1948). Expansion of trade led to a shift from silver to gold; in 1340 Lübeck made her gold coins based on the Florentine gold florin, *fiorino*, which functioned as blueprint, maintaining initially a steady and uniform denomination, while Flanders copied its gold from France, the *real* from the *écu* and the *mouton d'or* from the French one, but German monetary arrangements were messy (Shaw, 1895). The Hanseatic response was to create monetary unions. Monetary fragmentation led to treaties to establish shared measures of value; the Count of Flanders had established with his cities a shared value in the eleventh century, while the Wendish Monetary Union, *Wendische Münzverein*, was formed by Lübeck, Hamburg, and Wismar in 1379, while Rostock, Stralsund, and Lüneburg joined in 1381 (Jesse, 1927 [1967]). The aim of this union was to standardize minting, because the multitude of coins with different value for the same denomination hampered Hanseatic trade (Dollinger, 1998). However, this union only covered silver coins, thus excluding gold. Lübeck minted its own gold coin, *gulden*, and by using the Florentine florin *fiorino*, as

blueprint, Lübeck provided a hard international currency with restrictions to payments in gold (Shaw, 1895; Jesse, 1927 [1967]; Spufford, 1986; Dollinger, 1998). The Lübeck silver *schilling* depreciated to the Lübeck *gulden*, from less than ten *schilling* per *gulden* in the 1340s and 1350s to more than thirty from the 1450s up to the 1480s (Spufford, 1986). In such a bimetallist system, gold and silver coins were legal tender with a common unit of account, which started tied to a particular coin that over time might no longer be minted, thus becoming an abstract unit (Redish, 2000).

The money of account was not imaginary, according to Roover (1948), but could be based upon a real coin, such as the *groot* in Flanders. In Lübeck and the Wendish Monetary Union, Spufford (1986) points out that the non-minted Lübeck *mark* of 16 *schilling* was used. Both the *groot* and the *mark* represented a silver content of a particular purchasing power, but it was not always stable. Spufford (1986) finds that he *pond groot* depreciated to the *mark* from the 1340s to the 1390s, while it appreciated from the late 1390s to the 1470s. This can be attributed to changes in relative purchasing power. According to Roover (1948), inflation was high in fourteenth century Bruges, leading to the ordinance of 1389-90, invovlving a new *groot*. The relative purchasing power of the *pond groot* to the *mark* declined and a monetary adjustment led to a recovery of relative purchasing power lost. The gradual upward adjustment of gold, Redish (2000) explains by gold tending to be undervalued. However, the appreciation rate varied. By 1371 the ratio of Lübeck gulden to Rheingulden was 1:1, by 1450 the Lübeck gulden had appreciated to 3:2 (Spufford, 1986), suggesting that it was indeed a hard currency. This was important, because Bruges was a center for bills of exchange business, and bills of exchange were denoted in some fictitious unit, often gold that was stable, according to Einzig ([1962] 1970). Shortage of monetary metal and was solved by use of bills of exchange (Roover, 1948, Einzig,

1962 [1970]). Silver can be seen as the original heuristic, gradually including gold in some bimetallist heuristic.

The bills of exchange established an accounting system of exchange. This was achieved with the rise of exchange banks during the seventeenth century, in particular *Amsterdamsche Wisselbank*, established in 1609, but there were others in e.g. Hamburg, while *Stockholms Banco*, established in 1656, was a combined exchange and credit bank. *Amsterdamsche Wisselbank* as a 100 percent deposit bank, while *Stockholms Banco*, violated the 100 percent reserves criterion by issuing banknotes beyond actual deposits received in cash (Huerta de Soto, 2006).

The bill of exchange had emerged as a cashless means of payment of European merchants in the Middle Ages, but in Northwestern Europe and the Baltic Sea region, gradually adopted from the late sixteenth century, and the bill of exchange became a negotiable paper at the Antwerp stock exchange (Denzel, 2010). The creation of the *Amsterdamsche Wisselbank* meant a decisive step away from commodity money. Its *florin*, or *guilder*, was dominant as international currency in the late seventeenth and early eighteenth centuries (Gillard, 2004). However, the bank *guilder* existed only as balances in the accounts of the bank and in 1683 the balances with a claim with a 100 percent reserve ratio were supplemented with balances without a claim, irredeemable money, making the bank *guilder* more liquid than gold and silver coins, while negotiable receipts of deposit constituted fiat money (Quinn and Roberds, 2014). The receipt of deposit was an international success, which increased both the reserves of precious metals of the bank and the number of accounts, mainly due to correspondents of European merchants and bankers (Gillard, 2004).

Stockholms Banco issued in 1661 the first Western bank notes, the first ones outside China. Edvinsson (2010) stresses that Sweden had, during the 1624-1776 period has a complicated

multi-currency standard, copper and silver (and gold), including a fiat token standard, in 1716-1719, and a fiat paper standard, in 1745-1776, including alternation between copper and silver as main currency unit, depending on their relative price. According to Edvinsson (2012), Sweden had a trimetallic standard; copper, silver, and gold, involving two silver currencies, *riksdaler* for international exchange and *mark* for domestic exchange, and was characterized by the pre-modern monetary separation between units of account and means of payment, as abstract units of account two ghost monies were used; the *daler silvermynt* (dollar silver coin) and the *daler kopparmynt* (dollar copper coin). In 1644, large copper coins, the copper plates, started to be minted, to be deposited at exchange banks, denominated in the *daler silvermynt*, but not remaining at par with silver coins after the mid1650s, as their mint equivalent was increased (Edvinsson, 2010, 2012). Notes, *kreditivsedlar*, were issued in 1661, to avoid a bank run due to debasement, but were overissued (Wetterberg, 2009), were redeemed at par, but still traded at a discount at the market (Edvinsson, 2010). According to Edvinsson (2010), the fiat paper standard of 1745-1776 was an outcome of the *riksdaler* appreciating in 1741-1744 in *daler silvermynt* during a massive note issue due to a war with Russia, so the notes were made inconvertible into copper plates, while the gold coin *dukat*, issued during the 1654-1868 period, had a floating exchange rate to the *riksdaler*. Comparing the monies of account, the exchange rate of the Hamburg *mark banco* was stable to the Amsterdam *guilder banco*, while the Stockholm *daler silvermynt* depreciated and showed volatility (Denzel, 2010). Depreciation of copper coins to silver coins, the *mark kopparmynt* to the *riksdaler*, Sweden's international stable silver currency pegged to the Hamburg *reichsthaler banco*, increased the official rate from 6.5 in 1624 to 72 in 1777 (Edvinsson, 2010). Hence, an abstract unit of account, yet with an adequate metal base with 100 percent reserves, provided for stability, in contrast to multi-currency arrangements with fractional banking. An abstract unit of account, based upon bills of exchange, provided a suitable

heuristic, while a multi-currency standard made it hard to find a suitable heuristic until the international currency with a stable silver currency became the sole currency.

The gold standard united the unit of account and medium of exchange in a single asset, money, representing a specific amount of gold. The price of national currencies was fixed to a quantity of gold, by defining the price of gold for the currency at issue. The gold standard emerged in Britain in 1816-21, while France had a stable gold coin as of 1803, London and Paris taking over as Amsterdam had lost its role as the leading foreign exchange market. (Einzig [1962] 1970; Denzig, 2010). Convertibility into gold was crucial in uniting the unit of account and medium of exchange functions, because not only was the unit of account defined as a quantity of gold, but the medium of exchange had the same precise definition, being redeemable into gold. Denzig stresses that the classic gold standard was based on convertibility of gold, exchange rate stability, and balance of payments adjustments, the latter two being automatic, so exchange rates showed extensive stability around the gold parity. In contrast, Einzig argues that there were wide exchange movements even during the gold standard as well as complications due to the co-existence of silver and gold; stressing that few countries allowed free flow of gold; Great Britain, the United States, and the Netherlands.

Redish (2000) finds that in Britain, a bimetallic standard was abandoned for a gold standard by legislation in 1816, which suspended free coinage of silver and turned silver coins into convertible token coins, while for France and the other Latin Monetary Union members,[1] a transition was made from bimetallism to a limping gold standard, which suspended the coinage of five *franc* silver coins, but kept them as full legal tender, by 1878. Following unification,

[1] The Latin Monetary Union was formed in 1865 by Belgium, France, Italy, and Switzerland, originally being bimetallist, but eventually adapting a limping gold standard, where the five *franc* coin remained a legal tender together with the gold *franc*, which in addition was minted.

Germany adopted the gold standard in 1871, and demonetized silver in 1873 (Shaw, 1895). In 1873, Sweden and Denmark adapted the gold standard, following Germany, rather than joining the Latin Monetary Union, and created a common monetary system, the Scandinavian Currency Union with the *krona/krone* as unit of account, in 1873, joined by Norway in 1875 (Lobell, 2010). Einaudi (2000) considers the Latin Monetary Union more as a coinage agreement, as the union was limited to silver and gold coins, excluding bronze coins and bank notes, that lacked a common unit of account, but was an attempt to create a *franc*-based Western European currency. According to Crisp (1953), in Russia, Count Witte's monetary reform in 1897-99 established the gold standard and devalued the currency, introducing the gold *rouble* as one fifteenth of an *imperial*, the gold coin introduced in 1885 for international settlements, thus restoring the parity between the paper *rouble* and the gold *rouble*. However, money was able to expand beyond gold. Drummond (1976) points out that bank money expanded rapidly after the establishment of the gold standard in Russia. Hence, gold was only a lose constraint.

Bills of exchange played an important role in losing the constraints set by gold. Wicksell ([1906] 1966) argues, in light of his pure credit economy, that bills of exchange increases the virtual velocity of money and credit the velocity of money, so credit can replace gold, while Meulen (1934) considers bills of exchange to be the origin of an ideal system of credit, where an increasing amount of notes are issued on a given gold basis.. Nevertheless, as money was convertible into gold, the value of money was given by gold. Gold became the heuristic for the value of money, as unit of account and medium of exchange. Hence, the gold standard united the unit of account and medium of exchange functions in a single asset, money defined as a quantity of gold.

The interwar gold-exchange standard was an adaptation to bills of exchange and the credit economy, being more comprehensive than the classical gold standard, in explicitly recognizing private money, such as bills of exchange, as a part of total money. Eichengreen (1987) considers the gold-exchange standard a hybrid system, where countries were required to maintain gold convertibility of their currencies and free international gold flows, but they were allowed to hold reserves in foreign exchange. Eichengreen (2008) argues that the classical gold standard was supported by the set of economic and political circumstances specific to that time and place, not being there during the interwar period, when wages were more rigid; governments were more susceptible to pressure due to extension of the franchise, labor parties; and social spending. It had become more difficlt to have gold as a hard contraint, because money had to be able to expand to meet the needs of the real economy. He points out that the gold-exchange standard emerged as gold was augmented by foreign exchange, while the expanding world economy in 1924-29 required additional liquidity, but the great problem was that monetary gold became unevenly distributed.

World War I meant, according to Feinstein et al. (1997), a shock of economic restructuring due to reallocation of resources; economic consequences of the peace treaties with severe disturbances to trade and production when twelve new countries were created in Europe; the end of financial solidarity among the Allies; and reparations that crippled Germany until the Dawes Plan of 1924. The Baltic and North Seas region got some new independent nations: Estonia, Finland, Latvia, Lithuania, and Poland, involving fragmentation of into several currencies. Finland had already her own, having enjoyed monetary autonomy as a Russian grand principality, while the Free City of Danzig established its own currency. The restoration of the gold standard was not uniform. According to Feinstein et al. (1997), there were three paths inflationary paths

shaping parities (years of de facto restoration of the gold standard): hyperinflation and currency depreciation to less than 10 percent of pre-war parities (Germany in 1923, Poland in 1926); continued inflation after 1920 and currency depreciation to 10-30 percent of pre-war parities (Finland in 1924, Belgium in 1926); and finally controlled inflation after 1920: return to the pre-war parity (Sweden in 1922, the Netherlands in 1924, the United Kingdom in 1925, Denmark in 1926, Norway in 1928). Hence, the Scandinavian countries, Great Britain, and the Netherlands restored the gold standard at its pre-war parity, suggesting stable purchasing power of their currencies.

During the gold-exchange standard, Keynes (1930a) states that the demand for money is actually a demand for purchasing power, and brings in the price-level, as the price of a composite commodity to determine how many units of money is required per unit of purchasing power, but he points out that the pre-war parity relies on using a wholesale standard to assess general purchasing power rather than income share spent. In addition, Keynes points out that under an international currency system, such as gold, the central bank must focus on the external equilibrium and let the domestic economy adapt, because a standard other than the purchasing power of money itself must be preserved. Finally, Keynes adds securities, and considers how prices of securities may influence the purchasing power of money under consensus of opinion; rising in a “bear” market and falling in a “bull” market, reflecting the higher level of complexity in the interwar economic systems.

There had been a bifurcation in history between the British gold standard and the French gold *franc* with the Latin Monetary Union as a means to create an international gold *franc* currency, while the German adoption of the gold standard in 1871 paved the way for the formation of the Scandinavian Currency Union. World War I put an end to the classical gold standard. Sweden

ended convertibility and imposed a ban on the export of gold in 1914, Denmark and Norway followed in 1916, but Sweden and Great Britain, having experienced inflation followed by a sharp deflation in 1921-23, went back to a gold standard at the old parities in 1924 and 1925, respectively (Bohlin, 2010). the three Scandinavian economies were less synchronized during the interwar years, Sweden restored gold standard at prewar parity in 1924, while tight monetary policy eventually brought Denmark and Norway back to the prewar parity in 1927 and 1928, respectively, Sweden having pursued a tighter monetary policy already in 1920 (Klovland, 1998). This explains the termination of the Scandinavian Currency Union.

The birth of the gold-exchange standard was the Genoa Confeence in 1922, which in addition to gold as reserve, entitled central banks to hold foreign balances, bills, short-term securities, and other liquid resources as reserves, which required fixed exchange rates and allowed foreign reserves to be used, like gold, for international settlements (Eichengreen, 1987). This suggests a mixed heuristic of gold and foreign exchange with fixed exchange rates. Here, gold convertibility was crucial.

When inconvertibility occurred in 1931, foreign reserves were liquidated and money supply was constrained (Eichengreen, 1987). In effect, a loose constraint became tight. In effect, the international monetary system disintegrated into three blocs: the exchange control bloc around Germany; the sterling bloc around Great Britain; and the gold bloc around the United States and France (Echengreen, 2008; Feinstein et al., 1997). These blocs reflected the distribution of monetary gold, France and the United States holding 63 percent of central monetary gold, two other gold bloc countries Belgium and the Netherlands together as well as sterling bloc countries United Kingdom, including the Irish Free State, Sweden, Norway, Denmark, Portugal, Finland, Estonia together, both holding about 6½ percent each, while Germany held less than 2 percent

(Eichengreen, 1987).[2] This suggests gold still had an important role as heuristics. When Great Britain left the gold standard in 1931, the Scandinavian countries followed, allowing for depreciation to sterling before establishing a durable peg for the remainder of the interwar period (Klovland, 1998).

The gold-exchange standard indeed used gold as a heuristic for the value of money. Yet, we must recall it was not a measure of value, the purchasing power of money. White's (2012) proposal to restore the gold standard now, focuses upon finding the right parity. He points out the necessity of finding a parity close to the current market price of gold, referring to the deflation that followed the British restored gold standard at prewar parity in 1925. As we have seen, Sweden had created deflation to make the currency appreciate up to the prewar parity before adopting it, and we see a similar pattern for Denmark and Norway. White's proposal resembles the gold-exchange standard, but he tries to find an appropriate parity, essentially trying to avoid deflation (or inflation) by setting a parity at the current market price of gold. It has affinity with Fishers's (1913) compensated dollar scheme, which would vary the gold content of the dollar in ordet to keep its purchasing power of the commodity bundle constant.

3 DEVELOPING A THEORETICAL MODEL UISNG COMBINATORIAL GAMES

Highly abstract models, such as Schelling's (1969, 1971) checkerboard model of residential segregation, may be useful for understanding, thus having an epistemic value, when linked to clusters, being considered in the context of either a family of related models or the menu of possible explanations, according to Ylikoski and Aydinonat (2014). They find that, in spite of the checkerboard model's unrealistic assumptions, it has obtained a high status in the literature, as an

[2] Author's calculation based on Eichengreen's data. Eichengreen only mentions that France and the United States in 1932 held 63 percent of the central monetary gold in the world.

example of invisible-hand explations, a predecessor of agent-based models, a good explanation in the social sciences, and an explanation using social mechanisms. There are two reasons for this, according to Ylikoski and Aydinonat, namely that models come in clusters, exploring *what-if* questions, and that models are used to construct how-possible explanations (HPEs), exploring possible explanations, the actual explanation being a special case.

For the HPEs, Ylikoski and Aydinonat make a distinction between a empirical *possible causal scenarios* and theoretical *causal mechanism schemes*. *Possible causal scenarios*, Ylikoski and Aydinonat see as alternative historical scenarios how the *explanandum* fact could have come about, alternative causal histories, thus establishing *factual possibility*, the historical scenario being compatible with facts. *Causal mechanism schemes*, in contrast, Ylikoski and Aydinonat find to be what abstract theoretical models, such as the checkerboard model, do to establish *causal possibility*, meaning explanatory sufficicency, that scenarios be based on general causal knowledge, thus providing the skeleton for causal scenarios and studying the achievements of specified mechanisms. As case in point, they mention that Schelling's checkerboard model shows that removal of institutional segregation does not necessarily remove the segregation, a mechanism, which causal scenarios must take into account.

The *explandum*, of this paper would the set of combination of units of account and media of exchange during four historical monetary regimes. Monetary heuristics of these four historical monetary regimes would be *explanans*. *Factual possibility* would be established by being compatible with the stylized facts of these four historical regimes. *Causal possibility* is then established by means of a highly abstract theoretical model. A board game is highly abstract as explanantions of historical phenomena.

Here, combinatorial game theory may be useful, since it provides a theoretical tool to analyze checkerboard games Checkers, Go, and Konane , as Demaine and Hearn (2008) point out. These

are games without chance. Combinatorial game theory considers interaction among two players, Left and Right, playing alternating moves with perfect information, where the first player unable to move loses (see Conway, 1976; Berlekamp et al., 1982; Fenner & Rogers, 2015). These games are purely strategic, because they involve perfect information and excludes randomness, the last player able to move is the winner in normal play, making them finite, but there are loopy games that can be infinite, and they are either impartial, when have Left and Right have identical moves from the same position, or partizan otherwise (Demaine & Hearn, 2008).

Bearing in mind that our monetary heuristic gives meaning to money as a social institution, an idea of its worth as measure of value, winning a boardgame illustrates a better capability to make sense of settlements made due to the underlying heuristic, in other words a more useful heuristic. Rather then two market agents, say merchants or bankers, we have two different monetary heuristics of market agents. By this we mean rules of thumb that help them make sense of their economic environment. A way to do this would be free market auctions of a Go endgame, where there is strategic interaction that makes one particular heuristic or set of heuristics more effective, thus contributing to monetary stability.

Berlekamp (1996) starts by two important questions relevant to combinatorial games, such as Go and Domineering: (i) Who is ahead, and by how much? (ii) How big is the next move? According to Berlekamp, classical combinatorial game theory answers with *value* and *incentive*, while the ideal economist's view implies the game is a contest to accumulate points, eventually convertible into cash, thus monetizing the answers into prices, determined by competitive free-market auctions. This brings in monetary prices as means to signaling devices for communication.

In Go, the two players move by putting stones on vacant intersections on a 19 x 19 grid board, one player using black stones and the other player white stones. Hence, the choice of Left and Right becomes, Black and White, respectively.

Berlekamp presents the following procedure:

(i) The two players each submit a sealed bid, in order to play Black; the referee calculates their mean, μ; the player with highest bid plays Left (=Black) and pays the other player, Right (=White), the mean of the two bids, μ, which Berlekamp calls the *market value*, *mean value*, or in thermography the *mast value*.

(ii) Whenever making a move, each player pays a fee, or tax, t, to the other player: at each stage, the player can either pay and move or pass.

(iii) After two consecutive passes, a new auction is made with bids strictly less than the previous tax rate, but not below the minimum bid; a player may pass the auction, but the game ceases when both players pass the auction.

Berlekamp introduces so called Book values, being the fair market value, μ, and fair tax rate, the temperature, t; where the value of playing Left is μ, and the size of the move is t, because both Left and Right playing by the Book strategy means that Left assures a net gain being at least μ, while Right assures that gain to be at most μ. He then continues with thermography, as a method for deriving Book values of μ and t. This means some essential Go concepts need consideration:

(i) *Trajectory:* a linear combination of temperature t, and output value, v or μ; t plotted on the vertical axis, and v on the horizontal with increasing positive values to the Left and increasing negative values to the Right. A thermograph is a pair of trajectories, the Left wall, LW, and the the Right wall, RW; constructed from another pair of trajectories, the Left scaffold, LS, and the right scaffold, RS.

(ii) *Hills, caves, and masts:* for a given temperaure interval, *a hill* is a solid region, where LS > RS (Left scaffold exceeds Right scaffold), while LW = LS and RW = RS (walls and scaffolds coincide); a *cave* is a gaseous region, where LS < RS, while LW and RW coincide in a local trajectory, the *mast*. (Right scaffold exceeds Left scaffold); *normal mast*: vertical from the drain of the cave (the summit of the hill below) to the chimney of the cave (the fulcrum of the hill above). The thermograph is given by the walls, being equal to the scaffolds for hills and a vertical mast for caves. All hot games have thermographs with a hill at $t = 0$ and a vertical mast as t approaches infinity.

(iii) *Sente and Gote:* if an opponent's move increases the temperature, then it has *sente* and requires an immediate response, if both walls branch outward just below the mast, then the initial move by either player is *gote*, not requiring a response; if Left's (Right's) wall branch drops vertically, while Right's (Left's) wall branches outward, then Left's (Right's) initial move is *sente*, while Right's (Left's) move is *reverse sente*.

Sente means to have initiative, to force the other player to respond, playing defensive rather than offensive, to move to prevent the other from winning rather than moving to win, thus making a potential winning strategy wait.The economic meaning of Go may, however, be hard to assess. The beoard could be the set of potential exchanges and settlements. We could see that moves involve transaction costs, that there are strategies of evaluation and that there are attempts to exploit arbitrage possibilities, when commodities have not yet reached their correct market prices in units of account. Winning the Go game would be the end of the market process, when one heuristic prevents the other from conquering more settlements, as the opportunity space has

been fully exploited. The winning heuristic then provides an accurate unit of account, giving a stable exchange rate.

However, it may be suitable to study auctions by combinatorial games that involve elemtns of classical game theory, which is used in economics. Lazarus *et al*. (1999) develop a hybrid approach, which combines combinatorial game theory (Elwyn Berlekamp, John Conway) with classical game theory (John von Neumann, Oskar Morgenstern), to consider almost impartial games, according to which there are vertices such as sliding a token to them leads to an instant victory to a particular player, in addition to just let the first player who fails to move to a vertex loose along the arc of a directed graph. In particular, they consider a Richman game, which was developed by David Ross Richman:

(i) there are two players (Mr. Blue and Ms. Red) who each has some money;

(ii) Blue's goal is to bring the token to v_b on some directed graph and Red's goal is to bring the token to v_r, where v_b and v_r are the two distinguished vertices, vertex blue and vertex red, respectively, all other vertices are classified as a vertex black, which all have a path to either of the two distinguished vertices;

(iii) both players repeatedly bid for the right to make the next move by secretly writing a nonnegative real number no larger than the number of dollars he or she has, subsequetly revealed simultaneously.;

(iv) the highest bidder pays the amount of the winning bid to the opponent and moves the token from the vertex it currently occupies along an arc of the directed graph to a successor vertex;

(v) the game ends when one player moves the token to one of the two distinguished vertices, vertev blue or vertex red.

Here, there are no alternating moves, but the allocation of moves depends on bidding, thus making the distribution of money important. This would be of importance if you consider the medium of settlement used, say gold. The distribution of monetary gold would be essential to win.

Lazarus *et al*. (1999) point out that the probability of winning depends on the fraction of total money supply, based upon the following theorems, where $R(v)$ is the critical fraction of the total money supply that Blue needs to exceed in order to force a win, and $r(v)$ is the the critical fraction of the total money supply that Blue needs to exceed in order to avoid a loss:

(i) *Theorem 1:* There exists a Richman cost function, $R(v)$, for any directed graph D (not necessarily finite);

(ii) *Theorem 2:* Suppose Blue and Red play the Richman game on a (not necessarily finite) directed graph D with the token initially located at vertex v. If Blue's share of the total money supply exceeds $R(v) = \lim_{t\to\infty} R(v,t)$, then Blue has a winning strategy; his victory requires at most t moves, when his share of total total money supply exceeds $R(v,t)$. If Blue's share of the total money supply is less than $r(v) = \lim_{t\to\infty} r(v,t)$, then Red has a winning strategy; her victory requires at most t moves, when her share of total total money supply is less than $r(v,t)$. $R(v,t)$ and $r(v,t)$ are Richman cost functions.

(iii) *Theorem 3:* If the directed graph D is finite, then there is only one Richman cost function on D.

For finite games without cycles, Lazarus *et al.* point out that $r(v) = R(v)$, and find an optimal bid for both players: $\Delta = R(v) - R(v^-) = R(v^+) - R(v)$ times total money supply (coveniently set to one), where v is the current vertex, v^+ and v^- are a successor of v with maximal and minimal

Richman cost, respectively. In other words, winning requires a sufficiently high bid to move from the current point to the one that would yield a win. For this purpose, Lazarus *et al.* define a subgraph *D'*, which retains only edges (v,v^+) and (v,v^-), where v^+ and v^- are the successors of v with $R(v^+) = R^+(v)$ and $R(v^-) = R^-(v)$, and present two corollaries:

(iv) *Corollary 4:* Suppose Blue and Red are playing the Richman game on a finite directed graph D with the token initially located at vertex v. If Blue's share of the total money supply is less than $R(v) = \lim_{t\to\infty} R(v,t)$, then Red has a winning strategy;

(v) *Corollary 5:* Let D be a finite directed graph in which every vertex that is connected to v_b is also connected to v_r and vice versa. Then for all black vertices v, $R(v) = ½$; i.e., whoever begins with more money wins.

They point out, if both players play optimally, $R(v)$ is the chance that Red wins and $1 - R(v)$ is the chance that Blue wins, thus deviating from combinatorial game theory, which deals with games without chance. In games without chance, the games have a structure that determines who wins. However, from an exonomist's perspective, the distribution resources is of interest, say the distribution of gold during the gold standard and gold-exchange standard, thus leading us to exchange rates, which are an important aspect of monetary heuristics.

Lazarus *et al.* make an extension of the Richman game called Marksman, which introduces another currency, the deutsche mark, in addition to the US dollar, and thereby the exchange rate, ξ, the number of deutsche marks for a dollar. In this game, Herr Blau is substituted for Mr. Blue, and he bids number x, while Ms. Red bids number y; so $x > y$ implies that Blau pays Red x deutsche mark and $x < y$ implies Red pays Blau y US dollars. Instead of the Richman cost function with total money supply set to one US dollar:

$$R(v_b) = 0;\ R(v_r) = 1, \text{ for } v \text{ blue and red,}$$

$$R(v) = (R^+(v) + R^-(v))/2, \text{ for } v \text{ black},$$

where $R^+(v)$ and $R^-(v)$ denote the maximum and minimum values, respectively, Lazarus *et al.* introduce the Marksman cost function with total money supply set to ξ deutsche marks = 1 US dollar:

$$M(v_b) = 0;\ M(v_r) = 1, \text{ for } v \text{ blue and red},$$

$$M(v) = \text{avg}_M(M^+(v), M^-(v)) \text{ for } v \text{ black},$$

and the Marksman's average:

$$\text{avg}_M(x,y) = (\xi\, x + y)/(\xi + 1),$$

so Blau may win by bidding $\Delta = (M(v^+) - M(v^-))/(\xi + 1)$. They point out that the right to move is biased: Blau's chance is $1/(1 + \xi)$, while Red's chance is $\xi/(1 + \xi)$, so the probability of winning for the vertex v is the Marksman's average of the lowest and highest probability of winning among the vertices following v.

Hence, we observe that the Marksman's average reflects that numbers express equal worth regardless of currency, that is x and y have equal worth, thus presuming a shared underlying unit of account as measure of value for deutsche mark and US dollar, or at least a shared medium of settlement. We should consider the nature of numbers, thus opening for different meanings attributed by two players, using two different currencies.

4 MONEY AS A CLEARING DEVICE

We have to start with the meaning of numbers. According to Harper (2010), a number sequence is a yardstick, involving both an internal structure establishing a sequential ordering and an external structure relating to human intentionality and social activity, so the functioning of a

number sequence is social. He argues that number sequences are components of computational routines and number tools are parts of calculation processes, both involving human as well as non-human elements. Hence, we should open for the possibility that agents attribute different meanings for the same number. This implies that we cannot directly compare *x* and *y*.

In order to be comparable, they would need a shared medium of settlement, since money is a clearing device for the settlement of accounts. The two units of account used could be seen as fixed to the shared unit of settlement. We may use the discrete bidding approach of Develin, and Payne (2010), to outline a settlement game, in which bankers, who have different currencies, play to have the claims of their client merchants, by means of a shared medium of settlement, say merchant bankers clearing their accounts to complete bills of exchange transactions. The basic idea is that money as a clearing device owes its value to its purchasing power and that exchange rates reflect relative perceived purchasing power of currencies, where the purchasing power of money depends upon its efficiency in the settlement of accounts. When a monetary heuristic gives a less effective measure of value, it will lose, and the losing banker's currency depreciates relative to the one of the winning banker in a settlement of accounts setting.

Consider an indexed unit of account, whose purchasing power is expressed as an index, that is monetary unit with a stable purchasing power of a commodity bundle (Shiller 1998, 1999; Hall, 2005; Shubik, 2005). This idea goes back to Fishers's (1913) compensated dollar scheme, which would vary the gold content of the dollar in ordet to keep its purchasing power of the commodity bundle constant and would involve gold certificates called yellowbacks. The indexed unit of account in itself corresponds to Leijonhufvud's (1980) blueback scheme, by which the blueback would appreciate relative to the greenback with the same percentage as the greenbacks depreciate.

Having the unit of account functionally separated from the medium of exchange is the key chracteristic of 'new monetary economics', which involves monetary separation, splitting up money to separate the unit of account from the medium of exchange (see Hall, 1982a,b; Black, 1970; Fama, 1980; and Greenfield & Yeager, 1983). In particular, Greenfield & Yeager (1983) propose the Black-Fama-Hall (BFH) system, in which the government defines the unit of account based on a commodity bundle that is separate from media of exchange to be provided by private fund banks.

In contrast to their commodity bundle unit of account, various forms of abstract units of account have been proposed by Steuart (1810), Kitson (1895), and Meulen (1934). The abstract unit of account would be closer to Fisher's compensated dollar. Compare it with Meulen's (1934) banknote pound as invariable unit of value, which was to be created by substituting banknotes for gold coins and then letting the price of gold in banknote pounds fluctuate according to market conditions for gold. Meulen's banknote pound provides a stable abstract unit of account expressing the price of gold, a price that varies with the market purchasing power of gold, while Fisher's compensated dollar scheme adjusts the gold weight of the dollar to changes in a comprehensive price index. In either case, the purchasing power of gold varies rather than being constant. This is the idea of pre-modern monies of account, where some coin whose purchasing power is well known, such as *daler silvermynt*, is taken as the numeraire.

Following Fisher, Hall (2005) argues that a foolproof way to create a self-stabilized monetary unit is to define it in terms of units of a standardized security rather than a commodity bundle, mentioning the Unidad de Fomento (hereafter called UF), used in Chile since 1967, whose underlying resource is a bearer security of the Bank of Chile. The Amsterdam *guilder banco* could be considered as such a standardized security, as abstract unit of account.

Shiller (1998) calls the UF the first successful indexed unit of account, in which an exchange rate is established between this unit of account and the medium of exchange and store of value, the peso, by means of the consumer price index, as the UF is a lagged daily interpolation of the monthly consumer price index. In addition, Shiller makes a case for multiple units of account, actaually dual units; two units, one based on consumer price index, the other on an income index. In any case, a stable purchasing power is the defining criterion. The consumer price index and an income index are heuristics for a stable unit of account. Here we have bundles of commodities to define a measure of value with adjusting weights.

Another approach would be to use a bundle of currencies, an index for a global monetary constant, such as a constant unit of account, consisting of fifteen currencies, accounting for more than four-fifths of world GDP, accounting for inflation (Rahn, 2010), thus being an extension of the consumer price heuristic. Similarly, Ho (2002) proposes a World Currency Unit, a bundle of ”world output”, representing the purchasing power over a basket of about four-fifths of world GDP in the base year, represented by five economic zones: the United States, the Euro zone, Japan, Canada, and Australia, accounting both for inflation/deflation and currency depreciation/appreciation.

Rather than these designed global schemes, we would focus upon regional currency areas and emergent currency units within regions. The region at issue is the Baltic and North Seas region, and there are four historical institutional cases we will consider:

(i) H-money: The Hanseatic shift from silver to gold, comparing the Lübeck hard gold currency, the silver currency of the Wendishe Monetary Union (including Lübeck), and the silver currency of Bruges, considering the rise of bills of exchange;

(ii) X-money: The establishment of sixteenth century exchange banks and the use of bills of exchage to develop an accounting systems of exchange, comparing *Amsterdamsche Wisselbank* with *Stockholms Banco*, the stability of the Amsterdam bank money versus the depreciation of the domestic currency to the international one during Sweden's multi-currenncy standard, including fiat standards;

(iii) G-money: The gold standard, which united monetary functions into a single asset, money, when the price of national currencies was fixed to a quantity of gold and convertibility into gold was established, comparing the British gold standard and the initial bimetallism of the Latin Monetary Union, and Germany's adoption of the former, followed by the three Scandinavian countries, which formed the Scandinavian Currency Union, and Russia's late and quite flexible adoption;

(iv) I-money: The interwar monetary fragmentation, involving an augmentation of gold to include foreign reserves, that is the formation of the gold-exchange standard. Which meant a looser constraint, comparing different parities and break-up into three monetary blocs reflecting different paces to inconvertibility, which made the constraint tighter.

Behind them, there are exchanges and thereby settlements to be made, and having adequate resources to make these settlements. There are units of account with their own particular monetary heuristics, such as silver (and later gold) representing a specific purchasing power for H-money, while an abstract money of account, based upon bills of exchange, 100 percent reserves and adequate precious metal reserves, providing a suitable heuristic, unlike a multi-currency standard with fractional reserves made it hard to find a suitable heuristic under X-

money. Gold became a well-functioning heuristic due to convertibility into gold with G-money, but less effective with I-money when foreign exchange supplemented gold.

Turning to combinatorial game theory, using thermography, we may consider μ to be the equilibrium valuation of a commodity bundle, its correct price, while t is the transaction cost paid in units of account to make a settlement of a claim, a disequilibrium price premium paid. The move is to have the settlement completed. Let the Go board represent the opportunity space, each intersection being a commodity bundle having been exchanged.[3] The board itself may decomposed into separate subgames to illustrates local exchanges. Black (Left) and White (Right) represent two media of account, and thereby two different monetary heuristics. The market value is the equilibrium market value of a commodity bundle settled, while the temperature is the arbitrage value, arising from the possibility of exploiting profit opportunities. The concepts that would be relevant are the temperature of a game, its being hot (profit opportunities attracting agents with both heuristics, or cold, as well as *sente* and *gote*, the former implying initiative, forcing the other.

Another would be to use Richman games, incuding two currecies, possibly using a discrete Richman games, along the lines of Develin and Payne (2010). The latter would involve a total of claim tickets, the transaction cost to have a settlement done. The highest bidder gets the privilege to move thus completing the settlement. What would be important here is that the currency is stable for the agent having completed all settlements in the own unit of account, the winner of the game, but tunstable for the loser. Herr Blau and Ms. Red have a total of k claim tickets denominated as one unit of settlement, and the sum of their bids can never be greater than the

[3] The opportunity space may be augmented by, like Berlekamp does by e.g. playing a game that is the sum of Go and Domineering.

total number of claim tickets. In this context it would make sense to compare bids of ξx and y, where ξ changes as the outcome of the game.

The actual formalization in terms of combinatorial games remains to be done, but it would involve a winning path, for a heuristic, when the game reaches a particular point. The underlying idea is that the points would differ for different heuristics in different monetary arrangements.

5 CONCLUSION

Money is a social institution, representing some purchasing power, allowing for the settlement of accounts. As social institution, money has a cognitive dimension, which represents the way traders think about money as unit of account and medium of exchange, respectively, in the form of monetary heuristics, but money also has a behavioral dimension, which is expressed in the purchasing power of money. Exchange rates between currencies are established according to relative perceived purchasing power. It is therefore plausible to assume that actual exchange rates reflect monetary heuristics of market agents. Using stylized facts from four historical cases, this paper analyzes how to formalize monetary evolution as an ongoing loop between monetary heuristics and exchange rates. This paper outlines ways to formalize, and the formalization remains to be done.

The four historical regimes in the Baltic and North Seas region are considered: Hanseatic Monetary Arrangements; Seventeenth Century Exchange Banks; the Gold Standard and Monetary Unions; and Monetary Fragmentation and the Gold-Exchange Standard. The Hanseatic monetary arrangements were complicated, leading to formation of monetary unions, such as the Wendish one for silver coins, while Lübeck issued a gold coin that became an international hard currency due to its stability, that is a heuristic based on silver supplemented by gold. Seventeenth century exchange banks, starting with *Amsterdamsche Wisselbank*, established an accounting

system of exchange based upon bills of exchange, while the bank controlled trade in precious metal and maintained 100 percent reserves, thus making its bank money an international hard currency, while *Stockholms Banco*, had fractional reserves and introduced notes that it overissued, in system with a multi-metallic standard at times supplemented with paper standards, causing instability that made heuristics hard to develop. The gold standard established the gold heuristic and united the medium of account and medium of exchange functions in a single asset, money convertible into gold, yet with varieties, the German unification and adoption of the gold standard being decisive in establishing the British gold standard rather then the French bimetallist standard, while monetary unions, such as the Scandinavian Currency Union, were instrumental in achieving financial integration. Still, gold was still a loose constraint due to bills of exchange. The interwar restoration of the gold standard involved monetary fragmentation due to new countries having been formed and restoration at different parities, some at the prewar parity, others at lower parities, while gold was augmented with foreign reserves, making gold a looser constraint, but also leading to three separate blocs, reflecting the distribution of monetary gold and establishes different paces to inconvertibility, tightening the constraint again.

Formalization of the underlying processes behind this historical development would include some highly abstract model that would give a plausible explanation rather than a factual one. Abstract models, such as checkerboard models of segregation, provide a causal possibility, simplifying a very complex process. Combinatorial game theory, which is suitable to address boardgames, may provide such a causal possibility. Bearing in mind that our monetary heuristic gives meaning to money as a social institution, an idea of its worth as measure of value, winning a boardgame illustrates a better capability to make sense of settlements. Rather then two market agents, say merchants or bankers, we have two different monetary heuristics of market agents. A way to formalize this would be free market auctions of a Go endgame, where there is strategic

interaction that makes one particular heuristic or set of heuristics more effective, thus contributing to monetary stability. Another way would be to let biddg decide moves within the context of some extended Richman game, where a heuristic establishes paths to win, thus assuring monetary stability. The basic idea is that money as a clearing device owes its value to its purchasing power and that exchange rates reflect relative perceived purchasing power of currencies, while the purchasing power of money depends upon its efficiency in the settlement of accounts. It may be helpful to consider indexed units of account, where heuristics are give by an index, say a price index defining an underlying standardized security. It is essential that weights are continually adjusted in accordance with market conditions.

REFERENCES

Aoki, M. (2011). "Institutions as Cognitive Media between Strategic Interaction and Individual Beliefs." *Journal of Economic Behavior & Organization*, 79: 20-34.

Berlekamp, E. (1996). "The Economist's View of Combinatorial Games." In: R. Nowakowski (Ed.), Games of No Chance: Combinatorial Games, MSRI Publications, Vol. 29, Cambridge: Cambridge University Press, 365–405.

Berlekamp, E.R., Conway, J. H., & Guy, R. (1982). *Winning Ways for Your Mathematical Plays*. New York: Academic Press.

Black, F. (1970). "Banking and Interest Rates in a World without Money: The Effects of Uncontrolled Banking." *Journal of Bank Research*, 1: 9-20.

Bohlin, J. (2010). "From Appreciation to Depreciation – The Exchange Rate of the Swedish Krona, 1913-2008." In: R. Edvinsson, T. Jacobsson, and D. Waldenström (Eds.), *Historical Monetary and Financial Statistics for Sweden: Exchange Rates, Prices, and Wages, 1277-2008."* Stockholm: Sveriges Riksbank with Ekerlids Förlag, 340-411.

Clower, R. (1999). "From Markets to Money." *Ekonomia*, 3:1-13.

Conway, J. H. (1976). *On Numbers and Games*. New York: Academic Press, 1976.

Cowen, T. & Kroszner, R. (1994). *Explorations in the New Monetary Economics*. Oxford and Cambridge, MA: Basil Blackwell.

Crisp, O. (1953). "Russian Financial Policy and the Gold Standard at the End of the Nineteenth Century." *Economic History Review*, New Series, 6: 156-172.

Demaine, E. D. & Hearn, R. A. (2008). "Playing Games with Algorithms: Algorithmic Combinatorial Game Theory." *arXiv:cs/0106019v2 [cs.CC]* 22 Apr 2008.

Denzel, M. A. (2010). *Handbook of World Exchange Rates, 1590-1914*. Farnham and Burlington, VT: Ashgate.

Develin, M., & Payne, S. (2010). "Discrete Bidding Games." *Electronic Journal of Combinatorics*, 17: R85.

Dollinger, P. (1998). *Die Hanse* (including Dollinger, P. and A. Graβmann, "Zur hanseatische Geschichtsforschung 1960-1997"), Fifth Enlarged Edition, Stuttgart: Kröner.

Drummond, I. A. (1976). "The Russian Gold Standard, 1897-1914." *Journal of Economic History*, 36: 663-688.

Edvinsson, R. (2010). "The Multiple Currencies of Sweden-Finland 1534–1803." In: R. Edvinsson, T. Jacobsson, and D. Waldenström (Eds.), *Historical Monetary and Financial Statistics for Sweden: Exchange Rates, Prices, and Wages, 1277-2008."* Stockholm: Ekerlids Förlag & Sveriges Riksbank, 133-237.

Edvinsson, R. (2012). "Early Modern Copper Money: Multiple Currencies and Trimetallism in Sweden, 1624–1776." *European Review of Economic History*, 16: 408–429.

Eichengreen, B. (1987). *The Gold-Exchange Standard and the Great Depression*. National Bureau of Economic Research, Working Paper No. 2198.

Eichengreen, B. (2008). *Globalizing Capital: A History of the International Monetary System*. Second Edition. Princeton, NJ: Princeton University Press.

Einaudi, L. (1936). "Teoria della moneta immaginaria nel tempo de Carlomagno alla revoluzione francese." *Revista di storia economica*, 1: 1-35.

Einaudi, L. L. (2000). "From the Franc to the 'Europe': The Attempted Transformation of the Latin Monetary Union into a European Monetary Union, 1865-1873." *Economic History Review*, 53: 284-308.

Einzig, P. (1966). *Primitive Money in Its Ethnological, Historical, and Economic Aspects* (Second Edition, Revised and Enlarged). Pergamon, Oxford.

Einzig, P. ([1962] 1970). *The History of Foreign Exchange*, Second Edition. London: Macmillan.

Fama, E. (1980). "Banking in the Theory of Finance." *Journal of Monetary Economics*, 6: 39-57.

Fenner, S. A. & Rogers, J. (2015). "Combinatorial Game Complexity: An Introduction with Poset Games." *arXiv:1505.07416v2 [cs.CC]* 24 Jun 2015

Feinstein, C. H., Temin, P., & Toniolo G. (1997). *The European Economy between the Wars.* Oxford: Oxford University Press.

Fink, A. (2011). "Under What Conditions May Social Contracts Arise?" *Constitutional Political Economy*, 22: 173-190.

Fisher, I. (1913). "A Compensated Dollar," *Quarterly Journal of Economics* 27: 213-235.

Gell-Mann, M. (1994). "Complex Adaptive Systems." In: G. Cowan, D. Pines, and D. Meltzer (Eds.), *Complexity: Metaphors, Models and Reality*. Santa Fe Institute Studies in the Sciences of Complexity, Proc. Vol. XIX, 17-45.

Gigerenzer, G. (2008). *Rationality for Mortals: How People Cope with Uncertainty*. Oxford and New York: Oxford University Press.

Gillard, L. (2004). *La banque d'Amsterdam et le florin européen au temps de la République neerlandaise, 1610–1820*. Paris: L'École des hautes études en sciences sociales, Paris.

Greenfield, R. L., & Yeager, L. B. (1983). "A Laissez-Faire Approach to Monetary Stability." *Journal of Money, Credit, and Banking, 15*, 302-315. Reprinted in Yeager, L. B. (1997), *The Fluttering Veil: Essays in Monetary Disequilibrium*, G. Selgin (Ed.), Indianapolis, IN: Liberty Fund.

Hall, R. E. (1982a) "Monetary Trends in the United States and the United Kingdom: A Review from the Perspective of New Developments in Moetary Economics." *Journal of Economic Literature*, 20: 1552-1556.

Hall, R. E. (1982b) “Explorations in the Gold Standard and Related Policies for Stabilizing the Dollar.” In R. E. Hall (Ed.) *Inflation: Causes and Effects*, Chicago: University of Chicago Press, 111-22.

Hall, R. (2005). Controlling the Price Level. *American Journal of Economics and Sociology*, 64: 93-112.

Harper, D. A. (2010). “Numbers as a Cognitive and Social Technology: On the Nature of Conventional Number Sequences used in Economic Systems.” *Journal of Institutional Economics*, 6: 167-190.

Hayek, F. A. von (1952). *The Sensory Order: An Inquiry into the Foundations of Theoretical Psychology*. Chicago: University of Chicago Press.

Ho, L. S. (2002). *A WCU-Based Currency Board as a Sustainable Exchange Rate Regime*. Mimeo, Department of Economics & Center for Public Policy Studies, Lingnan University, Hong Kong.

Holland, J. H. (2006). “Studying Complex Adaptive Systems.” *Journal of Systems Science and Complexity*, 19: 1-8.

Honda, H. (2007). “Copper Coinage, Ruling Power, and Local Society in Medieval Japan.” *International Journal of Asian Studies*, 4: 225-240.

Huerta de Soto, J. (2006). *Money, Bank Credit, and Economic Cycles*, First English Edition (Tr. M. A. Stroup). Auburn: Ludwig von Mises Institute.

Jesse, W. (1927 [1967]). *Der Wendische Münzverein*, Braunschweig: Klinkhards & Biermann.

Keynes, J. M. (1930a). *A Treatise on Money, Volume I: The Pure Theory of Money*. London: Macmillan & Co.

Keynes, J. M. (1930b). *A Treatise on Money, Volume II: The Applied Theory of Money*. London: Macmillan & Co.

Kitson, A. (1895). *A Scientific Solution of the Money Question*. Boston: Arena.

Klovland, J. T. (1998). "Monetary Policy and Business Cycles in the Interwar Years: The Scandinavian Experience." *European Review of Economic History*, 2: 309-344.

Kohn, M. (2001a). "Trading Costs and the Pattern of Trade in Pre-Industrial Europe." Draft chapter of *The Origins of Western Economic Success: Commerce, Finance, and Government in Pre-Industrial Europe*. Dartmouth College, Department of Economics, Working paper 00-06, http://www.dartmouth.edu/~kohn/00-06.pdf

Kohn, M. (2001b). "The Expansion of Trade and the Development of European Industry to 1600." Draft chapter of *The Origins of Western Economic Success: Commerce, Finance, and Government in Pre-Industrial Europe*. Dartmouth College, Department of Economics, Working paper 00-20, http://www.dartmouth.edu/~kohn/00-20.pdf

Lazarus, A. J., Loeb, D. E., Propp, J. G., Stromquist, W. R., & Ullman, D. H. (1999). "Combinatorial Games under Auction Play." *Games and Economic Behavior*, 27: 229-264.

Leijonhufvud, A. (1980), "Theories of Stagflation." *Revue de l'association francaise de finance*, 1: 188-201. Reprinted in Leijonhufvud, A. (2000), *Macroeconomic Instability and Coordination. Selected Essays of Axel Leijonhufvud*. Cheltenham: Edward Elgar.

Lobell, H. (2010). "Foreign Exchange Rates 1804-1914." In: R. Edvinsson, T. Jacobsson, and D. Waldenström (Eds.), *Historical Monetary and Financial Statistics for Sweden: Exchange Rates, Prices, and Wages*, 2010, 1277-2008." Stockholm: Sveriges Riksbank with Ekerlids Förlag, 291-339.

Menger, C. (1892a). “Geld.” *Handwörterbuch der Staatswissenschaften* III, vgl. No. 24 (3rd edition (1909), *Handwörterbuch der Staatswissenschaften* IV, 555-610). Jena: Gustav Fischer.

Menger, C. (1892b). “On the Origin of Money.” *Economic Journal,* 2: 238-255.

Meulen, H. (1934). *Free Banking: An Outline of a Policy of Individualism*. London: Macmillan.

Mises, L. von ([1912] 1924). *Theorie des Geldes und der Umlaufsmittel*. Second Revised Edition. Munich andLeipzig: Duncker & Humblot.

Põder, K (2010). “Credible Commitment and Cartel: The Case of the Hansa Merchant in the Guild of Late Medieval Tallinn.” *Baltic Journal of Economics*, 10: 43-60.

Rahn, R. W. (2010). “A Constant Unit of Account.” *Cato Journal*, 30: 521-533.

Redish, A. (2000). *Bimetallism: An Economic and Historical Analysis*. Cambridge University Press.

Quinn, S., & Roberds, W. (2014). “How Amsterdam Got Fiat Money.” *Journal of Monetary Economics*, 66: 1-12.

Roover, R. de (1948). *Money, Banking, and Credit in Medieval Bruges: Italian Merchant-Bankers, Lombards, and Money Changers: A Study in the Origin of Banking*, Cambridge, MA: The Medieval Academy of America.

Schank, R. C., & Abelson, R. P. (1977), *Scripts, Plans, Goals, and Understanding*, Hillsdale, NJ: Lawrence Erbaum.

Schelling, T. C. (1969). “Models of Segregation.” *American Economic Review*, 59: 488–493.

Schelling, T. C. (1971). “Dynamic Models of Segregation.” *Journal of Mathematical Sociology*, 1: 143–186.

Schumpeter, J. A. (1917–18). “Das Sozialprodukt und die Rechenpfennige. Glossen und Beiträge zur Geldtheorie von heute”. *Archiv für Sozialwissenschaft und Sozialpolitik, 44*, 627-715. Reprinted in Schumpeter, J. A., (1952). *Aufsätze zur ökonomischen Theorie*. Tübingen: J. C. B. Mohr.

Schumpeter, J. A. (1970). *Das Wesen des Geldes*. Edited by F. K. Mann. Göttingen: Vandenhoeck & Ruprecht.

Shaw, W. A. (1985). *The History of Currency, 1252 to 1894: Being an Account of the Gold and Silver Monies and Monetary Standards of Europe and America, Together with an Examination of the Effects of Currency and Exchange Phenomena on Commercial and National Progress and Well-being*, London: Wilsons & Milne.

Shiller, R. J. (1998). *Indexed Units of Account: Theory and Assessment of Historical Experience*. National Bureau of Economic Research Working Paper No. 6356.

Shiller, R. J. (1999). *Designing Indexed Units of Account*. National Bureau of Economic Research Working Paper No. 7160.

Shubik, M. (2005). “Stable Prices, Money, and the Cost of Living.” *American Journal of Economics and Sociology*, 64: 117-124.

Spufford, P. (1986). *Handbook of Medieval Exchange*. London: Offices of the Royal Historical Society.

Steuart, J. (1810). *Principles of Banks and Banking of Money*. London: Davis.

Wetterberg, G. (2009). *Pengarna & makten. Riksbankens historia*. Stocholm: Atlantis & Sveriges Riksbank.

White, L. H. (2012). “Making the Transition to a New Gold Standard,” *Cato Journal*, 32: 411-21.

Wicksell, K. ([1906] 1966). *Föreläsningar i nationalekonomi, Häfte II: Om penningar och kredit*. Fifth Edition. Lund: Gleerups

Ylikoski, P. & Aydinonat, N. E. (2014). “Understanding with Theoretical Models.” *Journal of Economic Methodology*, 21: 19-36.