

# Emergence and Evolution of Property Rights: An Agent Based Model Perspective

Enrico Bertacchini, Ilaria Bertazzi, Elena Vallino<sup>†</sup>

Department of Economics and Statistics “Cognetti de Martiis”, University of Torino

## Abstract

The aim of this project is to develop an agent based simulation model in order to understand the emergence of property rights in spatial settings and analyze the structural dynamics of different property regimes like for example private and common property. The research agenda is structured as follows. We provide a review on the theoretical tool which have been applied to analyze the emergence and the evolution of property rights. We contend that agent based models (ABMs), through their focus on adaptive complex systems, integrate and foster the analytical capacity of traditional approaches in several directions. Subsequently we present an agent based model which is grounded on the work of Gintis (2007). Firstly, the model aims to replicate the original results in an ABM environment. Secondly, we aim to test the equilibria of property and anti-property in their robustness to the unpredictable elements introduced by the use of ABM. Thirdly, we analyze changes in agents' strategic choices after the introduction of heterogeneous formation of expectations and different space exploration patterns.

**Keywords** Property rights, agent based computational economics, complexity modeling, evolution of institutions

**JEL Codes** C63, K11, P48, Q15

## 1. Introduction

The aim of the paper is to provide a critical assessment on how complexity modeling, with particular reference to agent based simulation, may improve and extend the traditional analytical approaches on the origin and evolution of property rights. As property rights are recognized as a key institutional component of societies and economies in shaping resource use and exchange (North 1990), in the last decades there has been an increasing interest by several disciplines in understanding and explaining their emergence, evolution and influence on economic activities and agents' behavior.

As noted by Krier (2009), the several works addressing this issue can be classified into two main approaches, namely the economics of property rights and the evolutionary game models on contest behavior. The former applies a rather comparative static analysis to investigate how changes in relative resource values and transaction costs affect the optimality of property regimes and it has been used to explain the formation and development of even complex property rights arrangements in different historical phases. Conversely, evolutionary game models on contest behavior address the dynamics of rights evolution emerging from the repeated

<sup>†</sup>For correspondence, address Ilaria Bertazzi (ibertazz@unito.it).

interactions by agents. Within this approach property rights emerge as a stationary equilibrium based on the stable strategies of populations of agents, which can be eventually considered as emerging social norms. However, this approach still remains grounded on the assumptions of rational choice theory to model agents' behavior. Further, being based on the game theoretical framework, it barely help explaining the development of mixed property regimes, such as those enabling the interaction of private and common uses.

Crucially, while the two approaches provide complementary perspectives, their methodological foundations do not enable the development of a comprehensive analytical framework to explain the origin, structure and evolution of property regimes. As a result, we contend that complexity modeling might be able to integrate and foster the analytical capacity of the two traditional approaches. In particular, we explore the potential role of agent based simulations, which, through their focus on adaptive complex systems, refer to the computational study of economic and social processes modeled as dynamic systems of interacting agents who do not necessarily possess perfect rationality and information (Axelrod 1997).

Based on a burgeoning literature applying agent based modeling to explain the emergence and change of institutions, we propose a research agenda to analyze how and to what extent ABMs may complement the existent theories on the origin and evolution of property rights in several directions. Crucially, we argue that agent based simulation may help testing the application of behavioral rules, such as those deriving from cultural traits, which go beyond rational choice theory. Secondly, agent based models not only allow identifying stationary outcomes such as in evolutionary game models, but also enable a better understanding of the timing of evolutionary patterns. Finally, ABMs enable to highlight how the relative resource value and transaction costs affect property rights delineation through the structure of local interactions among agents. To support our claim and highlight the challenges in applying this approach to the study of property rights, we present an agent based model on the evolution of private property derived from the model by Gintis (2007). In particular, given to the differences between the two modeling approaches, we test the robustness of equilibria conditions obtained in the original model and then we show how the introduction of adaptive behavior and decentralized local interactions by agents add insights in explaining the emergence of property and anti-property regimes.

The paper is organized as follows: section 2 presents different approaches that address the investigation of the emergence and evolution of property rights in the literature. Section 3 describes the functioning of our agent-based model. Section 4 presents the results, while in Section 5 we drive conclusions and present the research agenda.

## **2 Approaches to explain the emergence of property rights**

Research on the formation and evolution of property rights has been one of the most stimulating developments for the last decades in social sciences and new institutional economics. As soon as the property rights approach came out as a response to extend the theory of production and exchange in economics (Furubotn & Pejovich 1972), the questions concerning how property rights originated and evolved have been of paramount relevance for scholars. According to Krier (2009), it is possible to identify two main approaches which have taken ground to respond to such questions, namely the economics of property rights and the evolutionary game models on contest behavior.

The former approach originated by Demsetz (1967), who first applied economic analysis to address the issue of the origin and evolution of property rights. The basic intuition in Demsetz's seminal work is that property rights emerge when the benefits of establishing those rights, and thus internalizing resource use externalities, exceed the cost. The models which express a Demsetzian legacy, regardless their sophistication, have a common denominator in applying a cost-benefit analytical framework to investigate how changes in relative resource values and transaction costs affect the optimality of property regimes. In such a framework property rights emerge in response to the magnitude of the economic incentives to define and delineate those rights over the resources. The relative resource value is generally a function of the scarcity of the asset, while transactions costs in devising exclusionary and governance rules to enforce property rights are mainly due to technology and the size of the group at stake (Ellickson 1993). Such models are particularly suited to explain the optimality of different types of property regimes and how those affect the incentives of owners to exert their property rights (Lueck & Miceli 2007). By contrast, concerning the evolutionary dynamics of property rights, Demsetzian models offer only a rather comparative static analysis of the institutional change and hardly provide a complete account of the evolutionary process.

The latter approach addressing the emergence of property rights has its roots on the development of evolutionary game models to explain animals' territorial behavior (Maher & Lott 2000, Kokko, López-Sepulcre & Morrell 2006). Given such evidence, the basic intuition proposed by Maynard Smith (1974) was that, modeling contest behavior as an evolutionary hawk-dove game, if a contest shows some form of asymmetry (e.g. first possession of a site) then the asymmetry is used as a conventional cue to settle it. More formally, it can be shown that a behavior such that "if owner play Hawk, if intruder play dove" (generally labeled as Bourgeois strategy) is an evolutionary stable strategy (ESS) superior to the simple Hawk and Dove strategies.

Social scientists have soon incorporated the insights developed by biologists and applied them to human behavior (Sugden 1989, Hafer 2006, Gintis 2007, Baker 2003). The main advantage of these models is that they conceive property rights formation as the elimination of costly conflict. Such theoretical treatments generally assume a Hobbesian "state of nature" and describe conditions under which cooperation (the absence of resource allocation toward conflict) is possible. With this perspective, these models are more effective in providing a theoretical explanation of the emergence and evolution of property rights based on the agents' evolving behavior.

However, as compared to Demsetzian models, one limitation of this approach is that it cannot account for anything beyond the definition of very simple property rules. Optimality of property regimes, such as the emergence of either private or common property or the interaction of private and common uses, cannot be generally analyzed because of the dependence on asymmetries that must be crude in order to be effective. More paradoxically, while the specification of the behavioral underpinnings of the Hawk, Dove, Bourgeois game allows us to determine the conditions under which a property equilibrium exist, asymmetric contest games may lead to an equal anti-property equilibrium where the agents' stable strategy is exactly the opposite (Maynard Smith 1974, Gintis 2007). Finally, although these models have proved to be consistent in taking into account how ecological conditions affect agents' behavior, the effects cannot be modeled at a micro level to analyze how the spatial heterogeneity of ecological conditions impact agents' local interactions.

Another tool which is used since relatively recent times in order to study the emergence of institutions such as property rights is that of agent based models (Gräbner 2016). "Agent" refers to entities that are able to perform autonomous actions within their environment and to communicate with other agents. They may

have a bounded representation of their environment and the decision making process may be based on satisfying goals and incoming information (Ferber 1999). One important feature of agent based models consists in the fact that the final output emerges from the interactions among the agents and between the agents and their environment. Therefore this tool is suitable to investigate how norms develop out of systems that express complexity features. Phenomena such as feedback mechanisms and system adaptation can be incorporated in the analysis (Gilbert & Terna 2000, Hare & Deadman 2004, Matthews, Gilbert, Roach, Polhill & Gotts 2007).

There exist already various examples of investigation on the emergence of property rights and access rules to resources using agent-based simulations. We identify two main streams of literature on this topic. The first one focuses on the emergence of individual property rights over resources, while the second one deals with the adoption of access and governance rules as a dynamic of socio-ecological systems.

The works belonging to the first group reproduce the emergence of individual property rights in a strict sense. Three of these works base the analysis on real-world situations, considering specific historical circumstances. Thebaud and Locatelli (2001) deal with the driftwood brought to the shore by storms on the coast of Yorkshire in England. Bowles & Choi (2003) deal with the establishment of the first agricultural societies in the human history. Kimbrough (2010) investigates the California Gold Rush of 1848/49. These papers share the fact that they start with a situation of open access toward a non-renewable resource, or of access rules which are not clearly shaped or defined. Subsequently, agents in the respective models develop strategies that lead to the persistence of specific individual access rules over the resources. Without any reference to historical occurrences, Flentge, Polani & Uthmann (2001), instead, explore the emergence and the consequences of a “possession norm” in a simulated society. One result of their work is that when ownership claims of other are respected, the probability of survival of the population is higher but agents face short-term disadvantages.

The works belonging to the second group deal with the analysis of the behavior of socio-ecological systems and they are linked with the discipline of ecology. They investigate how human behavior regarding the exploitation of renewable resources influences the state of the resource itself. In turn, they consider how the individuals or the groups of individuals react to the environmental situation. Therefore, reciprocal feedback human-environment are taken into account (Grimm 1999). As a result, while the first stream of works presented above more properly deals with the emergence of individual property rights on non-renewable resources, in this case the focus is rather on the emergence of governance rules for the exploitation of renewable natural commons at a sustainable rate. In this framework, the work of Bossel & Strobel (1978) is considered to be seminal. They aim to overcome two shortcomings of systems simulation studies for policy development: the failure to consider cognitive processes, and the neglect of normative criteria and its changes. Other works which are considered to be influential are Bosquet, Cambier & Morand (1994) and Lansing & Kremer (1993). The first develops an agent-based model of management of fisheries in the central Niger delta. The second is one of the first simulations about collective natural resource management and it is seminal since it provides a formal representation of self-governance. It is about traditional irrigation systems in Bali, Indonesia. It shows that simple bottom-up interactions of farmer groups at village level can lead to a good performance of a very complex large-scale irrigation system (Janssen 2007). Deadman & Gimblett (1994) develop a slightly different kind of model that deals with outdoor activities management in natural areas.

A sub-group of this second stream of works utilizes agent-based models in order to study the dynamics of collective actions for the management of common pool

resources and the emergence of institutions. Two important works in this field are Deadman, Schlager & Gimblett (2000) and Janssen & Ostrom (2006).

Considering the works analyzed, it is possible to highlight a number of potential advantages in adopting complexity modeling through agent based methods in the study of the emergence and evolution of property rights. Table 1 summarizes the main distinguishing features of this approach in relation with the other two traditional analytical models.

Table 1 – Comparison of theoretical approaches explaining property rights emergence and evolution

	<b>Agents' Behavior</b>	<b>Agents' interactions typology</b>	<b>Agents' Population Characteristics</b>	<b>Environmental variables</b>	<b>Time-path towards equilibrium</b>
<i>Demsetzian models</i>	Rational Choice Theory assumptions	None	Homogeneous	Exogenous	No
<i>Evolutionary game models on contest behavior</i>	Rational Choice Theory assumptions	Exogenous	Homogenous	Exogenous	No
<i>Agent Based Simulations</i>	Learning Imitation Mutation Transmission	Endogenous Local	Heterogeneous	Endogenous	Yes

First, agent-based models introduce behavioral rules which diverge from rational theory assumptions, such as simple utility maximization. Agents' may be framed to utilize social comparison, cultural norms which are inter-gene rationally transmitted, imitation, learning, or some other evidence that their previous behavior is no longer functional or, at least, is less functional than other behaviors the agent can perform (Moss 2001, Bowles & Choi 2003, Thebaud & Locatelli 2001). This enables to elaborate more complex and realistic explanations of property rights configurations, which may be sub-optimal from a strictly economic point, but endure insofar they are socially accepted by a community (Thebaud & Locatelli 2001).

As property rights are a social construct whose definition depends on agents' agreement, agent based simulations are also particularly suited to address how property rights emerge and evolve through agents' interactions. However, while in Demsetzian models such aspect is not taken into account and in evolutionary game models interaction is at most exogenously framed by assuming random pairing of agents, agent based simulations allow researchers to model endogenously such interactions depending on location choices of agents, spatial resource availability or rules preferential ties based on social networks (Vriend 2006).

Another important difference is that the two traditional approaches mainly rely on homogenous populations of one or few types of representative agents with exogenously defined environmental conditions while agent based models may introduce various degrees of heterogeneity with regards to the attributes of the agents and of the biophysical world they are situated in (Epstein & Axtell 1996, Squazzoni 2010). The given flexibility for modeling heterogeneity and change allows to create

credible counterfactuals to observe the impact of differences in resource and agents features on property rules (Kimbrough 2010). Crucially, this allows studying the resilience of the emerged property rights structure, which means observing whether and how a particular set of rules is able to absorb disturbances (Janssen & Ostrom 2006).

As for the evolutionary process addressing how agents' preferences, behavior and interaction lead to a given institutional equilibrium, traditional approaches only barely model the time-path towards an equilibrium state. Conversely, through ABM it is possible to observe the processes by which rules emerge, become established and enforced and therefore it is possible to draw considerations on the timing of system's changes. (Thebaud & Locatelli 2001, Bowles & Choi 2003, Janssen & Ostrom 2006).

In summary, while the two established approaches on property rights, namely Demsetzian models and evolutionary games on contest behavior, provide complementary perspectives but their methodological foundations do not enable the development of a comprehensive analytical framework, we contend that agent based methods may extend and integrate the explanatory power of both Demsetzian and evolutionary game models by incorporating the main insights and advantages of the two latter approaches to provide a more complexity-oriented explanation of the emergence and evolution of property rights.

### **3. An application to the evolution of private property**

Gintis (2007) develops a model loosely based on the Hawk-Dove-Bourgeois game and the War of Attrition, where an equilibrium emerges such that contestants do not attempt to seize and possessors fend off any such attempt. The crucial element that Gintis aims to reproduce is the influence of the endowment effect, the notion that people value a good that they possess more highly than the same good when they do not possess it. The consequent effect of endowment to the utility function is called "loss aversion", according to which agents are more sensitive to losses than they are to gains. If agents exhibit endowment effect over a resource, then property rights can be established. Importantly, enforcement of these rights is intended to be carried out by agents themselves in a disperse, decentralized normative environment. The endowment effect leads the current owner to be willing to expend more resources to protect his incumbency than an intruder will be willing to expend to expropriate the incumbent.

The model assumes that agents know the present value of incumbency and non-incumbency over a non divisible, renewable good. This situation explicitly involves loss aversion where, by the model specification, dis-utility of loss exceeds the fitness cost of loss. When an owner faces an intruder, the intruder computes the expected value of attempting to seize the resource, and the incumbent determines the expected value of defending incumbency. In plausible cases there is a range of values for which the intruder decides not to fight, and the incumbent decides to fight if challenged (Private Property or "Bourgeois" equilibrium, in the Hawk, Dove, Bourgeois game). In Gintis' model the level of resources devoted to a contest is endogenously determined by explicit modeling of the contest itself as a modified version of the War of Attrition; in such sub-game, the initial commitment of a level of resources to a contest must be ensured by the agent, so that the agent will continue to contest even when the costs of doing so exceed the benefits. Such modeling choice is due to a need for credibility when contest is about to happen. Such pre-commitment can be seen as a degree of loss aversion.

Results of Gintis' evolutionary game model determine the conditions for Private Property Equilibrium to hold (Theorem 1), which are pinpointed as to be the same conditions for Anti-Property Equilibrium; Theorem 1 distinguish those two equilibria from the conditions where migrant always fights for possession and an incumbent always contests. In Theorem 2, Gintis claims that, give the same conditions, Private Property Equilibrium exhibits higher mean payoff values than the Anti-property one.

### 3.1 Description of our model according to the ODD protocol

#### 1. Purpose

This model aims to extend the work of the Evolutionary Game Theory model designed by Gintis (2007). It is based on the simple "Hawk-Dove" game over the property of some valuable good (a "patch") in an evolutionary perspective. The purpose of the current work is manifold: first, to replicate the original results in an ABM environment, secondly, to explore the parameter space where the replication holds and therefor, to test the original equilibria, identified in the Evolutionary Stable Strategy perspective (called "property" and "anti-property"), in their robustness to the unpredictable elements introduced by the use of Agent-Based Modeling. In particular, given to the differences between the two modeling approaches, we show how the introduction of a more adaptive behavior (strategy are updated as the encounters between agents happen) and decentralized local interactions by agents add insights in explaining the emergence of property and anti-property regimes. Thirdly, we aim to extend the scope of the behavioral exploration of the original model by introducing localized, heterogeneous formation of expectations among agents, and different space exploration locally embedded, which are going to influence their strategic choice over the single games during the simulation runs.

#### 2. Entities, state variables, and scales

The model entities are two:

- Agents: individual in this model are all of the same type; they are heterogeneous in the status they assume over time regarding the ownership over a certain patch. If an agent owns a patch -the one on which it is on, defining itself as an *owner* or *incumbent*-, it has a positive payoff over this ownership in each time-step (*fertile\_gain* slider input). In order to begin this stream of revenue over ownership, payoffs are initially decreased by a fixed cost, *investment\_cost*. If they do not own a patch, they are defined as *migrant*, and its payoff is decreased each step by the cost of moving around to find a fertile patch, input by the slider *moving\_cost*.

Each agent assigns a value to the two possible statuses, being an *incumbent* or being a *migrant*. These values can be equal for everyone in the simulation when the expectations are computed based on global information over the environment (how many fertile patches in the world, and in particular how many non-occupied ones, how likely patches can lose their fertility status, and so on as defined below by the formulas of *IncumbentValue* and *MigrantValue*, see the "Adaptation" section).

Each agent is also endowed with a definition of its behavior for the occasion when they have to play Hawk-Dove game (which conditions will be defined below), since these strategies are defined in potential situation, every agent defines both: the strategy in the case it is in the *incumbent* status, and the strategy in the case it is in the *migrant* (better *intruder*, when a migrant challenges an incumbent for the ownership of a fertile patch, therefor engaging in a game) status. The two strategic definitions are based on the two values assigned to being an *incumbent* or a *migrant*, and are updated just in the moment when two agents encounter and one is owner.

- patches: the spacial units can have two statuses: fertile and non fertile. The total share of fertile patches is constant over time, but the single cell may change its status over time given a certain probability (input by a slider in the interface); for any patch

that switches its status from *fertile* to *non fertile*, there is a specular *non fertile* cell that becomes *fertile*, therefor keeping the total share constant. Fertile patches can be occupied (e.i. “have an owner”), or not; this does not affect the dynamics of patches fertility. Non fertile patches can not be owned.

The space is a torus of 32×32 patches, over which a number of agents, initially established in a range from 1 to one thousand, move. Each time-step is a unit of time where agents move, establish ownership over fertile, non-occupied patches and may engage in games of defense/challenge over the ownership of an already occupied patch.

### 3. Process overview and scheduling.

- Each time step the “go” procedure contain the following sub-procedures, in the presented order:
- create the sub-groups of non-owners and owners from the current status of ownership
- if expectations are common: compute the value of being Incumbent and Migrant and update them for every agent.
- To non-owners:
  - move to find a patch;
  - if, after movement, the patch on which they find themselves is fertile: do action (defined below, in the “Interaction” section)
  - if expectations are based on experience: update MigrantValue and IncumbentValue by the experience of still being migrant or becoming migrant for the patch one owned lost its fertility status.
- re-define the sub-groups of non-owners and owners from the updated status of ownership
- To owners:
  - if expectations are based on experience: update MigrantValue and IncumbentValue with the experience of being owner (adding the amount fertility\_gain, or b) or becoming migrant for the patch one owned lost its fertility status.
- Change the status of patches from fertile to non-fertile according to the probability. Change the status of previous owners whose patch lost fertility because of the ransom shock.
- re-define the sub-groups of non-owners and owners from the updated status of ownership

## 4. Design Concepts

### 4.1 Basic Principles

The model is inspired by the idea that Evolutionary Game Theory models can be extended with the use of Agent-Based modeling; in particular, an extensive literature of Games and Evolutionary games uses the basic scheme of “Hawk-Dove” game to represent strategic behavior over some good possible property, starting from Maynard Smith and Parker (1976), which develop the asymmetric “Bourgeois” solution for the game (Hawk strategy when Owner, Dove strategy when Non-Owner). In terms of EGT, the Hawk-Dove-Bourgeois game is taken by Gintis (2007) as contribution to

explaining the endowment effect with endogenously generated cost of defending property and contesting it, and endogenously generated values of the ownership.

In this ABM, we extend the work of Gintis by the possibilities opened with the localized, heterogeneous display of actions and expectations in simulation technique. We intend to study not only the replicability of the equilibria obtained in the Evolutionary Game Model, but also the existence of the dynamic trajectories that lead towards them, therefor their stability and robustness. We intend also to compare the results obtained with the general setting of information that produce the endogenously given value of ownership as designed by Gintis (homogeneous and equal for every agent, based on global information), to a localized, experience-based set of heterogeneous expectations, more close to the inclination for disperse information of Agent-Based modeling.

#### 4.2 Emergence

The interest of this model is to analyze condition (in terms of parameters range) and trajectories for the emergence of the so-called “property” and “anti-property” equilibria, where in the first the leading pair of strategies is *Hawk when owner* and *Dove when non-owner* (HD strategic behavior, or Bourgeois); in the second, the opposite holds (DH). Of course, the emergence of other stable points of population composition are of high interest, as well as unstable, out-of-equilibrium trajectories.

#### 4.3 Adaptation

Individual adapt their strategies over defending or not its own (possible) ownership and respecting or not others' ownership, based on the evaluation of the (expected) value of being an owner and being a migrant.

When these expectations are homogeneous, they are computed on the bases of the ones in Gintis' model, using previous period's values in a recurrent way to determine current ones.

*IncumbentValue<sub>t</sub>*

$$IV_t = \lambda \pi_C + (1 - \lambda) \left[ \text{probLoosingFertility} * MV_{(t-1)}(1 - c) + (1 - \text{probLoosingFertility}) (IV_{(t-1)} + b) \right]$$

where:

- $\lambda$  is the probability of being an incumbent who is challenged, that is calculated on the previous period.
- $\pi_C$  is the (mean) expected value of being an Incumbent (owner) whose patch is found by a migrant. It is based on:
  - the expectation of encountering a migrant who is Hawk (%MigH) multiplied by the
    - percentage (%IncH) of owners how play Hawk, therefor have 50-50% chance to win a contest (if lost, paying the damage k).
    - the remaining owners who play Dove (1-%IncH), therefor leave the patch (become migrant).
  - The expectation of encounter with a migrant who plays Dove (1-%migH), multiplied by the value of maintaining the ownership (still subject to the possibility that the patch loses its fertility by the shock).
- When not challenged, the incumbent can become migrant because patches loose fertility (probLoosingFertility), or
- stay Incumbent.

Similarly,

### *MigrantValue<sub>t</sub>*

$$MV_t = w \left[ IV_{(t-1)} (1 - v) \right] + ww \pi_U + (1 - w - ww) MV_{(t-1)} (1 - c)$$

- a migrant can find a fertile unoccupied patch with probability  $w$  (defined below), become Incumbent and pay the investment cost  $v$ .
- it can find a fertile occupied patch with probability  $ww$ , multiplied by the expected value of the encounter, as probability of finding an incumbent who is Hawk (%IncH), multiplied by the mean expectation for the population of migrants (as composed by Hawks and Doves); and probability of finding an incumbent who is Dove, therefor acquire the patch for the share of migrants who are Hawks (become incumbents themselves, without paying the investment cost), or stay migrant if they are Doves.

- $w = \frac{\text{num. of unoccupied } \wedge \text{ fertile patches}}{\text{num. of Fertile}}$  <sup>1</sup>

- $ww = \frac{\text{num. of occupied } \wedge \text{ fertile patches}}{\text{num. of Fertile}}$

$v$  is the initial cost (investment) to be made when a new patch is found;  $c$  is the moving cost for migrants,  $b$  is the gain from a fertile patch for its owner.

#### 4.4 Objectives

Agents evaluate the goodness of its own strategies pair for owner and non-owner with the expectation designed.

#### 4.5 Learning

Agents, just in the case of experience-based expectations, learn and update the value they assign to the two possible statuses they may assume (*Owner/Incumbent* or *Migrant*). They can be represented as learning how likely it is to find a fertile patch, how likely a patch lose its fertile status, how likely it is that property rights are challenged in a fight (game), and by the experience, they learn how much to value property and how it is worthy to fight to defend it when it is established/of worthy it is to challenge it when they intrude.

#### 4.6 Prediction

Prediction is implicit in the formation of value over being *Owner* or *Migrant*. This is because the evaluation of the statuses is made (both in the case of common evaluation based on global information and in the case of experience-based evaluation) in order to “predict” if it is worthy enough engaging in a “fight” for ownership (both when incumbent or intruder).

#### 4.7 Sensing

Agents sense the status of the patch they are in, and the presence of others on it too. (Further development of the model include a movement scheme that involves also sensing the neighborhood cells, in order to optimize the search for fertile, unoccupied patches.) They do not know, in any case, if the others are about to play Hawk or Dove.

#### 4.8 Interaction

When two agents find themselves in a fertile patch, and if one of those is owner, the procedure “action”, called by the code when a migrant agent finds a fertile patch (being it occupied or not), leads the migrant agent to the interaction part of the simulation in the following way (notice that the code checks for the agents on a patch to interact to be just two at a time):

<sup>1</sup> See appendix on discussion over the proposed formula for  $w$  and the one used by Gintis.

- ask the intruder to evaluate the outcome of choosing strategy “Hawk” versus the strategy “Dove” (which means stay migrant);
- ask the incumbent to evaluate the outcome of choosing strategy “Hawk”, defending its own property, versus “Dove”, which means become a migrant in case the other plays Hawk;
- Both those evaluations are in terms of expectations, give the current population composition of Hawk and Doves in migrant and owner status.
- if the intruder and the incumbent play “Hawk”: go to “game” procedure defined below
- if the intruder plays “Hawk” and the incumbent plays “Dove”: ownership is reversed and the previous owner becomes migrant.
- if the intruder plays “Dove”: status quo is maintained.
- The “game” procedure, for “Hawk vs. Hawk” situation is defined as follows:
- Owner wins 50%, keeping ownership status quo;
- Non-owner wins 50%, switching ownership statuses;

In this sense, the ownership do not guarantee any advantage of success for defending property. Extension of the model should include the asymmetric statuses of the two players.

#### 4.9 Stochasticity

The model contains two random elements:

- The probability of a patch to switch its status, in one sense or the other, is fixed by the parameter *probability\_of\_loosing\_fertility*.
- The movement of a *Migrant* agent is random over the entire space.

#### 4.10 Collectives

In the case of evaluation based on global information, the share of *migrants*, *owners*, in respect to the fertility share and the unoccupied share of patches, are the aggregations that directly affect individual behavior, along with the number of games (threaten to ownership) that happen. All those aggregate events are important also in the case of experience-based evaluations, but this effect is, of course, indirect; moreover, it is based on recursive formulas on the values of the previous round of the simulation.

#### 4.11 Observation

The output of the model is evaluated in terms of the share of population of agents that form pair of strategic behavior (owner and migrant strategies) such as: [Hawk-Hawk],[Hawk-Dove],[Dove-Hawk] and [Dove-Dove].

A stable population of [Hawk-Dove] is defined as “Property equilibrium”, or “Bourgeois” population. A stable population of [Dove-Hawk] is called “Anti-property equilibrium”. We control also for the values of being incumbent and migrant, the number of encounters and games in the last step, and the share of Owners who play Hawk and the share of Migrants who play Dove; the last measurement is, in the definition of Gintis, the actual “Bourgeois” equilibrium, since we only observe the actual strategies implemented by the agents, not the potential ones.

#### 5. Initialization

The parameters that are fully explored in the experimental design are:

- $c / moving\_cost$  in the range [0,1]
- $k / fight\_cost$  in the range [0,1]
- $v / investment\_cost$  in the range [0,1]

- $p\_fertile$ : the share of fertile patches in the world (fixed, as they change location, as in Gintis'), in the range  $[0,1]$
- $num\_turtles$ : the number of agents, in the range  $[50,1.000]$

Initially, the experimental design set half of agents with *OwnerStrategy* Hawk, the other half with Dove. The same for *MigrantStrategy*.

*IncumbentValue* of time “0” is set as  $b$  (the gain of ownership), the *MigrantValue* is  $(1 - c)$  (the moving cost). The gain from ownership  $b$  is parametrized to 1, so all the costs are in respect to full ownership gains.

### 7. Submodels

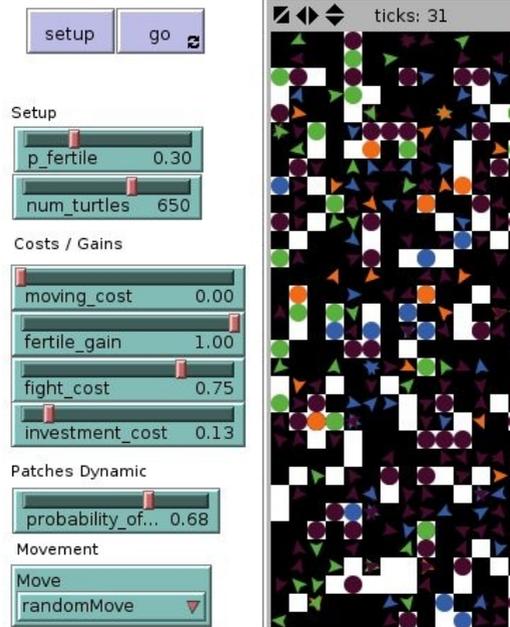
As described above, the model is intended to be tested in two modalities, or submodels: one with globally formed evaluations of being *Migrant* or *Incumbent*, and one in which those evaluations come from the experience of singular agents. The first step towards this heterogeneity is to build the values of *IncumbentValue* and *MigrantValue* not on the mean of the Incumbent and Migrant population, but specific to the actual strategies pair that an agent is endowed with at the moment they evaluate the statuses. Extension of the model include also submodels for different ways of movement across the space for migrant agents, more localized and possibly with some use of spatial information.

## 4 Results

The space of parameters that define the outcomes of our simulation is pretty vast, being 6-dimensional with the following possible intervals:

- $num\_turtles$   $[1 - 1000]$  (discrete)
- $p\_fertile$   $(0 - 1]$  (continuous, approximated with a discrete step of 0.01)
- $investment\_cost$   $[0 - 1]$  (continuous, approximated with a discrete step of 0.01)
- $fight\_cost$   $[0 - 1]$  (continuous, approximated with a discrete step of 0.01)
- $moving\_cost$   $[0 - 1]$  (continuous, approximated with a discrete step of 0.01)
- $probability\_of\_loosing\_fertility$   $[0 - 1]$  (continuous, approximated with a discrete step of 0.01)

Figure 1: Parameter set example in the Interface



In order to explore the effect of parameters combinations over strategic outcomes, and in particular the “Observable Behavior” of agents, we decided to perform two different explorations, using different techniques, each with a different goal. We define the “Observable Behavior” as the strategy adopted by the agent when in a specific status at the time of the observation. For example, if the agent is migrant at the current moment, its strategy as migrant is the observable one (not considering the “potential” strategy it would have performed if it was owner, even if such strategy is defined); symmetrically when the agent is owner on the time of observation, the owner strategy is the observable one. Therefore, the “Observable Status” of the simulation is constituted by the share of owners who have Dove/Hawk as observable strategy, and the share of migrants who have Dove/Hawk as current migrant strategy.

Such outcome analysis focuses on the global status of the population, and defines the emergence of “Property” as the situation where 100% of migrants respect incumbency of the others (have Dove strategy as migrant-strategy) and 100% of the current owner agents defend their property (perform Hawk strategic behavior as owner-strategy). At the opposite values stands the emergence of “Anti-Property”: 100% of migrant are Hawks, and 100% of owners are Doves.

As in any ABM, the first exploration of the parameter space is through unsystematic experimentation. It is common practice for researchers to experiment with different settings of their models, according to their intuitions. The results of our first exploration lead us to choose the following two deeper and more systematic exploration techniques, the first one having a more large spectrum of intent, the second being more specific to the target of defining the space of emergence of the “Property”, in continuity with the scope of this article.

#### 4.1 Exploration 1: Design of Experiment (DOE)

The first exploration we performed is based on the standard "Design of Experiments" (DOE) literature (Fisher, 1971), which consists in sampling points in

the parameter space in order to understand their effects.<sup>2</sup> If we intend agent-based simulation as an artificial laboratory, built in order to obtain in vitro representations of a (theoretical) social environment to be tested, such type of parameter space exploration is possibly the most immediate approach.

The scope of DOE approach is to infer some general trends in the whole space, therefor our design of Experiment is a complete exploration of the 6-dimensional space of parameters, with discrete steps for the continuous variables, with several repetition of simulation runs (ten) for the same parameters-combination, which allows to appreciate the importance / sensitivity of our model to the random elements present.

The points of sampling in the parameters that constitute our Design of Experiment are defined, for this stage, in the combination of the following parameters points:

Table 2: Parameters definition for the D.O.E.

[The experiment based on this DOE provide for  $(20 \cdot 4 \cdot 6 \cdot 6 \cdot 6 \cdot 6)$  points in the space, with 10 runs each, which leads to 777'600 runs]

Parameter	first	discrete step	final
num_turtles	50	50	1000
p_fertile	0.20	0.20	0.80
investment_cost	0.00	0.20	1.00
fight_cost	0.00	0.20	1.00
moving_cost	0.00	0.20	1.00
probability_of_loosing_fertility	0.00	0.20	1.00

We posed two possible conditions for the stop of the simulation,

- if the 100% of the agents in the simulation run adopt the same combination of strategies when owner or migrant, out of the four possible combinations [All agents are Hawk-Hawk, Hawk-Dove, Dove-Hawk or Dove-Dove]
- if the simulation reached one thousand ticks.

The goal of this exploration technique is to individuate the more stable areas of parameter space, the ones that produce unique outcomes (if they exists). Given this goal, the results are analyzed just for the cases of stop because of the first condition. The second one constitutes an upper-bound for the simulation in order not to run forever.<sup>3</sup>

This approach does not allow for a straightforward representation of the results due to the non linear effects that emerge from the complex interaction of the 6 dimensions of the model. Nevertheless, it allows to test the consistency of the original hypotheses and to identify some peculiar regularities. Firstly, we can appreciate that, when Property and Anty-Property regimes emerged, not only the “Observed Behavior” is defined as before, but the whole population of agents adopts respectively a

<sup>2</sup> Some examples of experimental designs include the factorial design (which tries all levels of all factors), the Latin hypercube design (which guarantees a certain degree of representativeness while sparsely sampling the space), and the sphere-packing design (which attempts to efficiently cover the space while sampling few points.)

<sup>3</sup>In our investigation, those results are not analyzed because they constitute an out-of equilibrium.

“Bourgeois” (Hawk-when-owner and Dove-when-migrant) or “Anti-Bourgeois” (vice versa) strategy combination.

Secondly, we grouped the results in the four different possible outcomes of population composition that we may observe. Consistently with the results of the Gintis' (2007) model, we never observe as final outcome a population adopting Dove strategy in any agents' status. Thirdly, we test the replication of our results to the ones in Gintis (2007) by testing whether its first theorem holds for our final outcomes:

**Theorem 1.**

If

$$IncumbentValue > (1 + fightCost) * MigrantValue * (1 - movingCost)$$

there is a unique equilibrium in which a migrant always fights for possession and an incumbent always contests. (100% HH)

When the reverse inequality holds, there exists both a private property equilibrium and an anti-private property equilibrium. (100% DH or 100% HD)

[Gintis, 2007, p. 9, variable names according to the present paper.]

In all our final states, being “Hostile” or “Property”/“Anti-Property”, the inequality of Theorem 1 holds, making our model a good replication of Gintis' Evolutionary Game model in an Agent-Based environment.

As for peculiar regularities, we find that:

- The “Hostile” equilibrium is possible over the entire range of parameters.
- On the contrary, Property and Anti-Property equilibria are never obtained when *moving\_cost* is greater than 0.4 and *fight\_cost* is smaller than 0.2. Such observation is consistent with Theorem 1; the values of being Incumbent and Migrant are endogenous to the simulation, but the inequality of theorem 1 is satisfied in any case if those two parameters are in their boundary levels, therefore leading to 100% HH equilibrium.
- The Property equilibrium is never observed when the probability of losing fertility is equal to zero, whereas the Anti-Property one is observed; one possible explanation for this regularity is to consider that when no turnover of fertility happens there is no other way for migrants to gain property over occupied patches, but to challenge the current owners.

#### 4.2 Exploration 2: Genetic Algorithms with *BehaviorSearch*

The second technique is based on specific target-outcomes. The question here is not, as previously, an overall observation of the behavior of the simulation but, instead, is how can we obtain the target outcome in the space of parameters, or, rephrasing, which combinations of parameters values produce the outcome we aim to obtain, for instance the Property and Anti-Property equilibria. The Genetic Algorithm (GA) approach adopted here focuses on searching for specific points in the space in order to answer questions about model behavior.

For this inquiry, we choose to adopt a tool of NetLogo software called “BehaviorSearch”<sup>4</sup> which uses Genetic Algorithms (and possibly other heuristic techniques) to explore the parameters space. Beyond the single software adopted, the technique called “Genetic Algorithm” belongs to the family of evolution-inspired algorithms (Holland, 1975). GAs are conceived as an application of principles of biological evolution to computer science, software design and also simulations in general. GAs belong to a group of methodologies developed in order to explore the parameter space called “Search methods”, where the modelers design an objective

4 Software and relative documentation is available at: [www.behaviorsearch.org](http://www.behaviorsearch.org)

function that expresses the characteristic behavior to be obtained. Genetic Algorithms design over a software “evolve” solutions to challenging problems by artificially mimicking the forces of mutation and natural selection on a virtual population of candidate solutions. Genetic Algorithms are a target-oriented search mechanism that has proven to be successful in a variety of combinatorial optimization and search problems.

There is a wide possibility of optimization and search techniques that have been developed through history of hard sciences. The choice, by the *BehaviorSearch* designer, of Genetic Algorithms is justified because they possess several characteristics that are useful for the domain of ABMs.

*<<First, genetic algorithms are a meta-heuristic technique that is general enough to handle the mix of Boolean, integer, discrete, continuous, and categorical parameters that may be present in agent-based models. Second, the objective functions are almost always stochastic and may be non-convex and genetic algorithms have often proven effective at escaping local optima in the search space [...], as well as progressing towards a goal despite noisy environments [...]. Third, the choice of genetic algorithms is motivated by an intuition that the crossover operator will be able to take advantage of partial solutions to speed the search process.>> (Stonedahl, 2011 p. 53-54)*

In general, Search Algorithms provide an answer to the question of “what settings of the parameters (in this case of the ABM) will result in the greatest/closest/majority expression of the behavior we aim to reproduce?” This is called the Query-Based Model Exploration (QBME), and can be summarized in the flow provided by the *BehaviorSearch* author. (Stonedahl, 2011, p. 76)

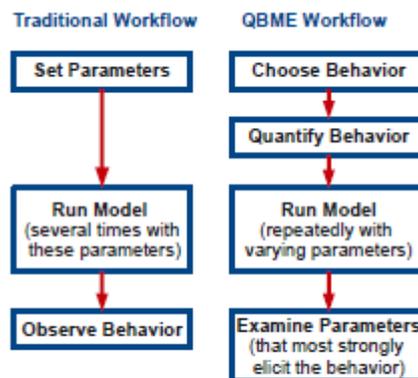


Figure 3.1. Flowchart highlighting the difference between the QBME paradigm and the traditional paradigm for model exploration.

Focusing on our model, and the macroscopic behaviors we target as objective function for the Genetic Algorithm, we choose as target the emergence of the “Property equilibrium” (individuals follow the rule: Hawk behavior if owner, Dove behavior if migrant).<sup>56</sup>

We obtained, as results of this Genetic Algorithm search, several points in the 6-dimensional space of parameters. Such points produce, again, instances of situations

5 Specification on the setting of *BehaviorSearch* can be found in the Appendix.

6 Specification on the setting of *BehaviorSearch* can be found in the Appendix.

where the 100% HD strategies combination is obtained as outcome. The characteristic of the results are than analyzed in the model by testing for each point:

- the persistence of the outcome to random elements of the simulation, and
- the stability of the resulting outcome to small perturbations; this is analyzed by seeing if there exists a convergence towards the same or similar outcome in points that are very close to the original ones ( $\pm 5\%$ ).

Afterward, we determine, based on the previous two analysis, the relationship between the points and their characteristics, and therefor we formulate hypotheses on the functioning of our model. Notice, again, that the particular nature of the AB modeling, due to non-linearity, does not guarantee the stability or convergence of results, therefor we cannot claim that the points we identify constitute the complete set in the entire space of parameters that perform the outcome we are interested in. This claim is reinforced by the results of the previously presented D.O.E.

The points obtained searching for Property equilibrium are:

Table 3: G.A. Result Points in the Parameter Space

#	p_fertile	num_turtles	Population pressure <sup>7</sup>	moving_cost	investment_cost	fight_cost	probability_of_loosing_fertility
1	0.30	650	2.12	0.00	0.13	0.75	0.68
2	0.37	1000	2.64	0.02	0.14	0.67	0.56
3	0.30	750	2.44	0.02	0.27	0.97	0.30
4	0.82	50	0.06	0.01	0.16	0.43	0.88
5	0.33	50	0.15	0.07	0.12	0.96	0.48
6	0.81	500	0.6	0.06	0.21	0.85	0.29
7	0.32	50	0.15	0.04	0.19	0.49	0.22
8	0.10	200	1.95	0.00	0.18	0.36	0.28
9	0.82	650	0.77	0.13	0.08	0.58	0.98
10	0.77	700	0.89	0.06	0.00	0.89	0.68

#### RUNNING THE POINTS IN THE MODEL

Interestingly, as shown in Table 4, all the points identified by the GA, when run several times on the model, display a bifurcation behavior; which are consistent with the model we aim to reproduce. They can both lead to a “Property equilibrium”, 100% HD strategies combination (the one that was the target of the optimization process) and the opposite “Anti-Property”. No other final population composition is reached, and different frequency of the two possible outcomes are observed, according to some characteristics of the parameters level. Moreover, it is worth to mention that the Incumbent and Migrant values that are obtained when simulation is run in such points are never higher (in mean values) when comparing the Property and the Anti-property equilibria.

Table 4: Share of 100%HD and 100%DH Outcomes in 1000 Replications of the above defined Points

#	100% HD (share)	100% DH (share)
---	-----------------	-----------------

$$^7 \text{ Turtles per fertile patch} = \text{num\_turtles} / (32 * 32 * \text{p\_fertile})$$

1	48.3%	51.7%
2	64.9%	35.1%
3	45.7%	54.3%
4	5.5%	94.5%
5	48.5%	51.5%
6	35.8%	64.2%
7	33.3%	66.6%
8	56.8%	43.2%
9	45.6%	54.4%
10	26.4%	73.6%

The determination of which of the two outcomes will emerge is not settled by the parameters (as the Evolutionary Game Theory approach suggests), but is the result of a path-dependent process as defined by Arthur (1989):

*<< [...] the process is non-ergodic or path-dependent -it is determined by its small-event history.>> (Arthur, 1989, p. 122)*

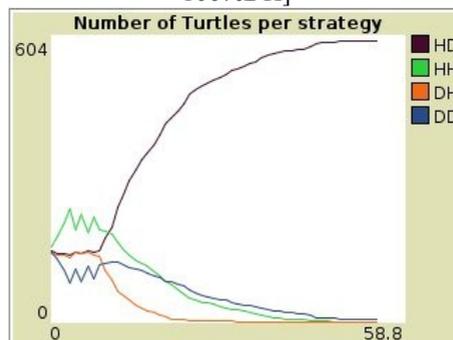
*<<Insignificant circumstances become magnified by positive feedback to 'tip' the system into the actual outcome 'selected'. The small events of history become important.>> (ibidem, p. 127)*

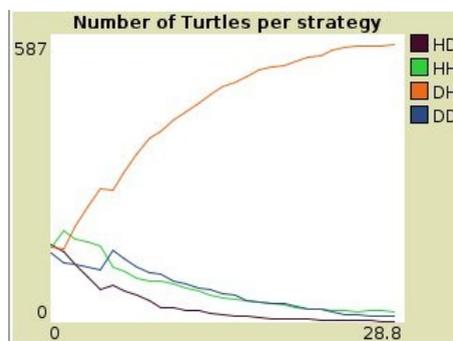
By "small events, insignificant circumstances, Arthur intended:

*<<I therefore define 'historical small events' to be those events or conditions that are outside the ex-ante knowledge of the observer -beyond the resolving power of his 'model' or abstraction of the situation.>> (ibidem, p. 118)*

In our case, the "small events" are the random elements of the model, like the initial position of the agents and the fertile patches, and the sequence of encounters between agents. Those elements are the ones that create path-dependency in the history of our model, leading to bifurcation of possible, completely opposite outcomes (see Figure 2a and 2b).

Figures 2a and 2b: Plots of Share of Population Dynamics Under the same Parameter Setting (#1). The first simulation lead to 100%HD, the second to 100%DH]





Given the sampling size of our experiment (one thousand runs) over the pinpointed positions, we cannot statistically exclude the distribution of 50% results of one type and 50% of the other for points #1,#3, #5, #8, #9. Experiments with parameters values at Point #4 lead to a large majority of "Anti-Property" Equilibrium outcomes, and it appears intuitive to ascribe such difference to the low pressure of population versus the great amount of fertile patches present in such setting. The same reasoning, but in smaller terms, applies to points #6, #7 and #10. Point #2 has a slight majority of "Property" Equilibrium outcomes, and here the population pressure over fertile patches seems to be high enough to justify the result. Notice that, even if #5 and #7 have the same population pressure, the distribution of results is not the same. In this difference we appreciate the importance of other parameters, such as, in this case, the cost of fighting, which is very high in #5, leading to a lower occurrence of contesting behavior and therefore a larger share of "Property" Equilibrium outcomes. Similarly, when comparing points #6 and #9, we see the relative importance of the higher moving cost of #9, that drives results towards more equal distribution of HD and DH outcomes in respect to the previous setting.

In respect to such distributions, and the above mentioned dynamical path-dependency, we can provide a perspective to the selection of equilibrium in respect to the one provided by Gintis. In case of possible presence of Property and Anti-Property equilibrium Gintis claims,:

<<**Theorem 2.** (...) Then the anti-private property equilibrium exhibits a lower average payoff than the private property equilibrium. (...)

Theorem 2 helps to explain why we rarely see anti-property equilibria in the real world, If two groups differ only in that one plays the private property equilibrium and the other plays the anti-private property equilibrium, the former will grow faster and hence displace the latter, provided that there is some scarcity of resources leading to a limitation on the combined size of the two groups.>> (Gintis, 2007, p. 12-13)

The ABM approach we choose to adopt in this paper, on the other hand, allows us to assume a different perspective in terms of explanation of the selection among the two possible equilibria. In theoretical terms, both the EGT perspective (“*given the same conditions, property equilibrium provides higher payoffs than the anti-property one, thus will be selected more frequently*”) and the ABM one (“*the selection is given by small, unpredictable elements in a path-dependent dynamics, paired with the general condition of the environment, for example the relative scarcity*”) provide valuable explanations.

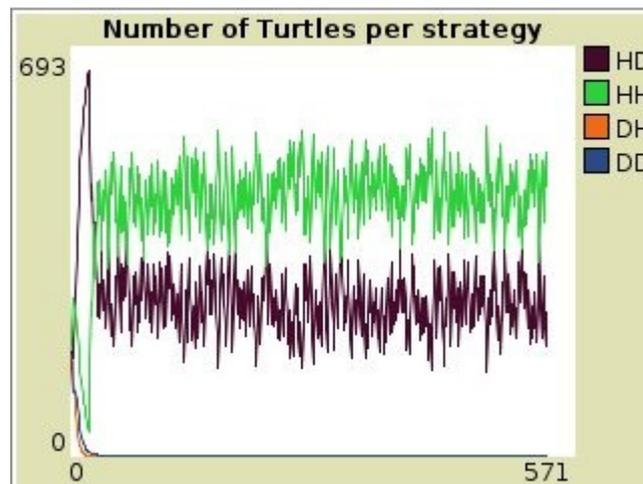
#### ROBUSTNESS OF RESULTS TO SMALL PERTURBATIONS

We performed a stability/convergence check on the pinpointed parameters levels by increasing and decreasing them of 5% (or smaller amounts if the parameters

boundaries were reached)<sup>8</sup>. From such effects due to changes in value of single parameters, we can appreciate that, generally, the most sensitive one for the desired output is the cost of movement (*movement\_cost*) when increased. This is consistent with the D.O.E results, in which we excluded high level of *movement\_cost*. Point #9 displays also sensitivity to a decrease of *fight\_cost*, again consistent with D.O.E. results that excluded low levels of this parameter. Points #7 and #9 also present sensitivity to higher level of *investment\_cost*; the behavior of *investment\_cost* appears in migrant value in case of found fertile unoccupied patch weighted by *w* (probability to find unoccupied fertile patches).

In many cases, as displayed in Figure 3, we observe a particular type of deviation from the bifurcation results of Property and Anti-Property Equilibrium. Such deviation displays out-of-equilibrium long-run behavior, producing a population composition that oscillates in Hawk-Doves and Hawk-Hawk, meaning that: for the “owner” status, defense is the dominant strategy in any case; for “migrant” status, the ecology of the population share requires the con-presence of both Bourgeois agents (HD) and more aggressive ones (HH). Notice that the same parameters produce also an Anti-Property equilibrium outcomes.

Figure 3: Plot of Population Share Dynamics in case of Out-of-Equilibrium Long Run results.



This result allows to highlight the importance of the Agent-Based modeling approach to describe results that go beyond the study of equilibrium, and that get closer to a realistic depiction of the emergence of private property regimes in different populations (where, for example, properties are defended by owners but not always respected by others).

## 5. Discussion and conclusions.

The paper has proposed a research agenda to analyze the scope of agent based simulations to explain and model the formation and evolution of property rights. While the two established approaches addressing this subject, namely Demsetzian models and evolutionary games on contest behavior, provide complementary perspectives but their methodological foundations do not enable the development of a

<sup>8</sup> An overview of the effects obtained in such exploration is in Annex 2

comprehensive analytical framework, we contend and discuss how complexity modeling can represent a step forward to integrate the explanatory capabilities of the two approaches in several directions. As a first step, we present an application of ABMs to replicate the theoretical model proposed by Gintis (2007) to explain the evolution of private property.

Currently, in our model agents' heterogeneity is defined according to ownership status, the time sequence of events such as finding fertile patches when migrant, being challenged or the patch loosing fertility when owner. In order to perform the most accurate reproduction of Gintis' framework, agents are modeled to have uniform values over the two possible ownership status.

The preliminary results obtained so far already confirm the potential in applying ABM to investigate the emergence and evolution of property regimes. In particular, going further into the research agenda, the next steps will entail higher levels heterogeneity among agents to further explore the explanatory possibilities.

The first step towards building heterogeneity among agents is in the formation of ownership and non ownership values: starting from the current uniform setting, we can easily implement agents that evaluate the value of their statuses based on their current strategy combination. How such modification will affect the emergence and stability of the outcomes we previously presented is an important element in terms of validity of the general model of Hawk/Dove representation of private property emergence.

Another step consists in a design of localized information and strategic movements. On one side, we can test the localized restriction of the knowledge of the environment, both in terms of fertility share and composition of the population of other agents (instead of the global information of share of agents behavior and patches occupied and fertile). On the other side, we can build more strategic agents with smart movements as related to the goal of establishing ownership over, firstly, unoccupied fertile patches. Such modification creates a more strict spatial embedding of the phenomena that the model represents, but also it constitutes an improvement of agents behavior in terms of realism, adding strategic reasoning also to the action of moving. Such implementation can be accompanied with different costs associated with different steps, allowing for even more strategic choice over, for example "how far to move".

Lastly, but possibly more interestingly, we aim to add forms of learning for the agents in the model, by evaluating experience of one strategic behavior or the other (therefor having completely heterogeneous evaluation of the values of migration and ownership), and also by learning the composition and state of the environment (not only by locally perceiving it, but also adjusting such perception in time). If the stability of property and anti-property equilibria is ensured even with the introduction of trials-and-errors procedures and memory construction, then such more realistic setting could constitute a big step forward in the explanatory power of the models that aim to target the existence of private property in human societies and in (certain) animals.

## References

- Anderson, C. L. & Swimmer, E. (1997), 'Some empirical evidence on property rights of first peoples', *Journal of Economic Behavior & Organization* **33**(1), 1–22.
- Anderson, T. L. & Hill, P. J. (1975), 'The evolution of property rights: A study of the American west', *Journal of Law and Economics* **18**.
- Gräbner, C. (2016). 'Agent-based computational models—a formal heuristic for institutionalist pattern modeling?' *Journal of Institutional Economics* **12**, 241–261.
- Axelrod, R. M. (1997), *The complexity of cooperation: Agent-based models of competition and collaboration*, Princeton University Press.
- Baker, M. J. (2003), 'An equilibrium conflict model of land tenure in hunter-gatherer societies', *Journal of Political Economy* **111**(1), 124–173.
- Bosquet, F., Cambier, C. & Morand, P. (1994), 'Distributed artificial intelligence and object-oriented modeling of a fishery', *Mathematical and Computer Modeling* **20**(8), 97–107.
- Bossel, H. & Strobel, M. (1978), 'Experiments with an “intelligent” world model', *Futures* **10**(3), 191–212.
- Bowles, S. & Choi, J.-K. (2003), The first property rights revolution, in 'Workshop on the Co-evolution of Behaviors and Institutions, Santa Fe Institute'.
- Brian Arthur (1989) Competing technologies, increasing returns, and lock-in by historical events, *The Economic Journal* **99**, pp. 116–131.
- Clay, K. & Wright, G. (2005), 'Order without law? property rights during the California gold rush', *Explorations in Economic History* **42**(2), 155–183.
- Deadman, P. & Gimblett, R. H. (1994), 'A role for goal-oriented autonomous agents in modeling people-environment interactions in forest recreation', *Mathematical and Computer Modeling* **20**(8), 121–133.
- Deadman, P. J., Schlager, E. & Gimblett, R. (2000), 'Simulating common pool resource management experiments with adaptive agents employing alternate communication routines', *Journal of Artificial Societies and Social Simulation* **3**(2), 2.
- Demsetz, H. (1967), 'Toward a theory of property rights', *The American Economic Review* **57**(2), 347–359.
- Ellickson, R. C. (1993), 'Property in land', *Yale Law Journal* **102**(6), 1315–1400.
- Epstein, J. M. & Axtell, R. (1996), *Growing artificial societies: social science from the bottom up*, The MIT Press.
- Ferber, J. (1999), *Multi-agent systems: an introduction to distributed artificial intelligence*, Vol. 1, Addison-Wesley Reading.
- Field, B. C. (1989), 'The evolution of property rights', *Kyklos* **42**, 319–345.
- Fisher, R. A. (1971). *The Design of Experiments*. Hafner.
- Flentge, F., Polani, D. & Uthmann, T. (2001), 'Modeling the emergence of possession norms using memes', *Journal of Artificial Societies and Social Simulation* **4**(4).
- Furubotn, E. G. & Pejovich, S. (1972), 'Property rights and economic theory: a survey of recent literature', *Journal of Economic Literature* **10**(4), 1137–1162.
- Gilbert, N. & Terna, P. (2000), 'How to build and use agent-based models in social science', *Mind & Society* **1**, 57–72.
- Gintis, H. (2007), 'The evolution of private property', *Journal of Economic Behavior & Organization* **64**(1), 1–16.
- Grimm, V. (1999), 'Ten years of individual-based modeling in ecology: what have we learned and what could we learn in the future? ', *Ecological modeling* **115**(2), 129–148.
- Hafer, C. (2006), 'On the origins of property rights: Conflict and production in the state of nature', *The Review of Economic Studies* **73**(1), 119–143.

- Hare, M. & Deadman, P. (2004), 'Further towards a taxonomy of agent-based simulation models in environmental management', *Mathematics and computers in simulation* **64**(1), 25–40.
- Holland, J. H. (1975). *Adaptation in Natural and Artificial Systems*. MIT Press.
- Janssen, M. A. (2007), 'Coordination in irrigation systems: an analysis of the lansing-kremer model of bali', *Agricultural Systems* **93**(1), 170–190.
- Janssen, M. A. & Jager, W. (2002), 'Stimulating diffusion of green products', *Journal of Evolutionary Economics* **12**(3), 283–306.
- Janssen, M. A. & Ostrom, E. (2006), 'Adoption of a new regulation for the governance of common-pool resources by a heterogeneous population', *Inequality, Cooperation, and Environmental Sustainability* pp. 60–96.
- Kimbrough, E. O. (2010), 'Invasion, self-defense and third-party enforcement: Modeling emergent property rights with applications to economic history'.
- Kokko, H., López-Sepulcre, A. & Morrell, L. J. (2006), 'From hawks and doves to self-consistent games of territorial behavior', *The American Naturalist* **167**(6), 901–912.
- Krier, J. E. (2009), 'Evolutionary theory and the origin of property rights', *Cornell Law Review* **95**(1), 139–160.
- Lansing, J. S. & Kremer, J. N. (1993), 'Emergent properties of Balinese water temple networks: Coadaptation on a rugged fitness landscape', *American Anthropologist* **95**(1), 97–114.
- Levmore, S. (2002), 'Two stories about the evolution of property rights', *Journal of Legal Studies* **31**.
- Libecap, G. (1994), *Contracting for Property Rights*, Cambridge University Press.
- Lueck, D. (1994), 'Common property as an egalitarian share contract', *Journal of Economic Behavior & Organization* **25**(1), 93–108.
- Lueck, D. & Miceli, T. (2007), Property law, in S. Shavell & A. M. Polinsky, eds, 'Handbook of Law & Economics', Elsevier.
- Maher, C. R. & Lott, D. F. (2000), 'A review of ecological determinants of territoriality within vertebrate species', *The American Midland Naturalist* **143**(1), 1–29.
- Matthews, R. B., Gilbert, N. G., Roach, A., Polhill, J. G. & Gotts, N. M. (2007), 'Agent-based land-use models: a review of applications', *Landscape Ecology* **22**(10), 1447–1459.
- Maynard Smith, J. (1974), 'The theory of games and the evolution of animal conflicts', *Journal of theoretical biology* **47**(1), 209–221.
- Miner, D. (2010). A framework for predicting and controlling system-level properties of agent-based models. Unpublished doctoral dissertation, University of Maryland Baltimore County.
- Moss, S. (2001), 'Game theory: limitations and an alternative', *Journal of artificial societies and social simulation* **4**(2).
- North, D. C. (1990), *Institutions, Institutional Change and Economic Performance*, Cambridge University Press.
- Sanchez, S. M., & Lucas, T. W. (2002). Exploring the world of agent-based simulations: simple models, complex analysis. In WSC '02: Proceedings of the 34th conference on winter simulation (pp. 116-126).
- Santos JI, Pereda M, Zurro D, Álvarez M, Caro J, Galán JM, et al. (2015) Effect of Resource Spatial Correlation and Hunter-Fisher-Gatherer Mobility on Social Cooperation in Tierra del Fuego. PLoS ONE 10(4): e0121888.
- Smith, H. E. (2002), 'Exclusion versus governance: Two strategies for delineating property rights', *Journal of Legal Studies* **31**, 453–87.
- Squazzoni, F. (2010), 'The impact of agent-based models in the social sciences after 15 years of incursions', *History of Economic Ideas* **18**(2), 197.

- Stonedahl, F. (2011) Doctoral Thesis. Genetic Algorithms for the Exploration of Parameter Spaces in Agent Based Models. Northwestern University.
- Sugden, R. (1989), 'Spontaneous order', *The Journal of Economic Perspectives* 3(4), 85–97.
- Thebaud, O. & Locatelli, B. (2001), 'Modeling the emergence of resource-sharing conventions: an agent-based approach', *Journal of Artificial Societies and Social Simulation* 4(2), 3.
- Van Beers, W. C. M., & Kleijnen, J. P. C. (2008). Customized sequential designs for random simulation experiments: Kriging metamodeling and bootstrapping. *European Journal of Operational Research*, 186 (3), pp. 1099-1113.
- Vriend, N. J. (2006) 'ACE models of endogenous interactions'. In Testfatsion, L. & Judd, K.L., *Handbook of Computational Economics*, 2, 1047-1079. North Holland
- Young, H. P. (2006), 'Social dynamics: Theory and applications'. In Testfatsion, L. & Judd, K.L., *Handbook of Computational Economics*, 2, 1081-1108. North Holland

APPENDIX 1: DETERMINATION OF  $w$  and  $ww$  (probability of finding a fertile unoccupied patch and probability of finding a fertile occupied patch)

According to Gintis,  $\omega$ , the probability of finding a fertile unoccupied patch, is defined as:

$$\omega = r * (1 - \alpha * \phi) \quad (\text{eq. 7})$$

Instead, we decided to formulate such variable as:

$$\text{prob. find FERTILE UNOCCUPIED patch} = \text{prob. find FERITILE} * (\text{prob. find UNOCCUPIED})$$

Or, in terms of conditioned probability:

$$P(F \cap U) = P(F) * P(U | F)$$

Notice that the probability of being Fertile is not independent from the probability of being Unoccupied, therefor we cannot say that  $= P(F)*P(U)$

So:

$$P(F \cap U) = P(F) * [1 - P(O | F)] \quad \text{because} \quad P(U | F) + P(O | F) = 1$$

Now,  $P(F) = r$  can be defined as  $f * p$ , in which the first term is share of fertile patches (constant in time) and the second is the probability of finding a patch (of any kind, we can safely assume to be 1, unless we impose some "out of space" temporary state).

Regarding  $P(O|F)$

$$P(O | F) = \frac{P(O \cap F)}{P(F)}$$

Gintis uses  $P(O)$  instead of  $P(O|F)$ , in fact:

$$P(O) = \frac{\text{num. of incumbent}}{\text{num. of patches}} = \frac{\text{num. of agents}}{\text{num. of patches}} * \left( \frac{\text{num. of incumbent}}{\text{num. of agents}} \right)$$

$$P(O) = \frac{n_a}{n_p} * (\text{share of incumbent})$$

It is possible that this mistake comes from the idea that the ABSOLUTE NUMBER OF OCCUPIED PATCHES EQUALS THE ABSOLUTE NUMBER OF FERTILE, OCCUPIED PATCHES. (since only the fertile ones can be occupied). But this is not true any more when talking about percentage, therefor probabilities.

More easy, in our model we can define:

$$P(O | F) = \frac{\text{num. of Occupied}}{\text{num. of Fertile}}$$

or, equivalently,

$$P(O | F) = \frac{\text{num. of incumbent}}{\text{num. of Fertile}}$$

but we can do it straight forward with the computation of:

$$w = P(U | F) = \frac{\text{num. of unoccupied } \wedge \text{ fertile patches}}{\text{num. of Fertile patches}}$$

Similarly, we can determine  $ww$  (probability of finding an occupied fertile patch) as:

$$ww = P(O | F) = \frac{\text{num. of occupied } \wedge \text{ fertile patches}}{\text{num. of Fertile patches}} = \frac{\text{num. of owners}}{\text{num. of Fertile patches}}$$

## APPENDIX 2: BEHAVIORSEARCH SETTINGS

Among other settings, we focused on the results coming from the following Genetic Algorithm setting in *BehaviorSearch*:

[Figure 4: BehaviorSearch Interface]

The screenshot displays the BehaviorSearch interface, which is divided into several sections for configuring a Genetic Algorithm search.

- Parameter Specification:** A text area containing a list of parameters and their ranges, such as `["p_fertile" [0 0.01 1]]`, `["num_turtles" [50 50 1000]]`, and `["moving_cost" [0 0.01 1]]`. A button below it reads "Load param ranges from model interface".
- Setup:** A dropdown menu set to "setup".
- Step:** A dropdown menu set to "go".
- Measure:** A text field containing the expression `(count turtles with [ownerstr = 0 and nonownerstr = 1]) / num_turtles`.
- Measure If:** A text field set to "true".
- Stop If:** A text field containing the condition `p_fertile * 32 * 32 >= num_turtles and p_fertile = 0`.
- Step Limit:** A text field set to "1000" model steps.
- Search Method Configuration:** A dropdown menu set to "StandardGA". Below it is a table:

Parameter	Value
mutation-rate	0.01
population-size	50
crossover-rate	0.7
population-model	generational
tournament-size	3

Below the table is a checkbox for "Use fitness caching" which is checked. A "Search Encoding Representation" dropdown is set to "StandardBinaryChromosome".
- Objective / Fitness Function:** A dropdown menu set to "Maximize Fitness".
  - Collected measure:** A dropdown menu set to "AT\_FINAL\_STEP".
  - Fixed Sampling:** A dropdown menu set to "Fixed Sampling" with a text field for "10" replicates.
  - Combine replicates:** A dropdown menu set to "MAX".
  - Options for "Take derivative?" and "Use ABS value?" are present but unchecked.
  - w.r.t.:** A dropdown menu set to "----" with a  $\Delta = 0.0$  field.
  - Evaluation limit:** A text field set to "1000" model runs.
  - BestChecking replicates:** A text field set to "0".

A "Run BehaviorSearch" button is located at the bottom right of the interface.

We imposed a possible exploration of the parameter space as complete as possible ("Parameter Specification" section), the objective function, to be maximized, is the share of agents behaving as Bourgeois (Hawk-Dove strategy pair). We therefore choose to compute the fitness of a generation of simulation settings by looking at the maximum value of the objective function obtained in such generation. Setting of the Genetic Algorithm are in the "Search Method Configuration", and gave good results with 3-generations GAs, each composed of 50 specification of settings, a low mutation rate of 0.01 and a crossover rate of 0.7.

APPENDIX 3: CONVERGENCE / STABILITY CHECK.

[TAB: EFFECTS OF 1-DIMENSIONAL PERTURBATIONS ON THE SELECTED POINTS IN THE 6-DIMENSIONAL PARAMETER SPACE. Difference of  $\pm 5\%$ , unless parameters' limit reached; when results do not change qualitatively, //]

#	$\pm p\_fe$ rtile	$\pm num\_$ turtles	$\pm moving\_cost$	$\pm investmen$ t\_cost	$\pm fight\_cost$	$\pm probabilit$ y\_of\_loosi ng\_fertility
1	//	//	+ 5% ALL HH <sup>9</sup>	//	//	//
2	//	//	-2% no effects +5% SOME DH, SOME NON-EQUILIBRIUM OF HH&HD COMPRESSENCE <sup>9</sup>	//	//	//
3	//	//	-2% no effects +5% SOME DH, SOME NON-EQUILIBRIUM OF HH&HD COMPRESSENCE <sup>9</sup>	//	//	//
4	//	//	//	//	//	//
5	//	//	//	//	//	//
6	//	//	//	//	//	//
7	//	//	-4% no effects +5% ALL HH <sup>9</sup>	-5% no effects +5% SOME DH, SOME NON- EQUILIBR IUM OF HH&HD COMPRES SENCE <sup>9</sup>	//	//
8	//	//	+ 5% ALL HH <sup>9</sup>	//	//	//
9	//	//	- 5% no effects + 5% SOME DH, SOME NON-EQUILIBRIUM OF HH&HD COMPRESSENCE <sup>9</sup>	- 5% no effects + 5% SOME DH, SOME NON- EQUILIBR IUM OF HH&HD COMPRES SENCE <sup>9</sup>	- 5% SOME DH, SOME NON- EQUILIBR IUM OF HH&HD COMPRES SENCE <sup>9</sup> + 5% no effects	//
10	//	//	//	//	//	

<sup>9</sup> The same applies when one or more of the other parameters vary too.