Property Rights Enforcement with Unverifiable Incomes

Jan U. Auerbach*

University of Exeter

March 21, 2016

Abstract

I study a planner's choice of the level of property rights enforcement when agents that are heterogeneous in productivity can appropriate each others' resources. The planner cannot verify incomes but hiding resources from taxation is costly. While this friction imposes a binding constraint, the planner implements perfectly secure property rights. The allocation incentivizes production through redistributive taxation and absorbs potential appropriators as personnel in enforcement. Higher costs of hiding income allow the planner to implement more redistributive taxation and less enforcement, leading to more production and higher welfare. A political economy friction does generate imperfectly secure property rights.

Keywords: Economic institution, property rights.
JEL classification: O17, P14.

*PRELIMINARY. Comments are welcome. E-mail: *j.auerbach@exeter.ac.uk*. Mail: University of Exeter Business School, Streatham Court, Rennes Drive, Exeter EX4 4ST, United Kingdom. I thank Costas Azariadis for many discussions. I have benefited from comments by seminar and conference participants at various occasions. Errors are mine.

1 Introduction

A society looking to protect property rights needs to commit designated resources. Private information on incomes (as well as potential distortions of the payoff structure in the economy) complicates the collection of these resources from its members through income taxation. It thus constrains society in its effort to raise funds to enforce property rights and may affect its choice of a property rights enforcement regime. So, does this complication help explain imperfectly and unequally secure property rights across countries? I ask what a planner can achieve when she cannot verify the income of those she would like to tax in order to finance enforcement; and to what extent variations in the severity of this friction may contribute to differences in the security of property rights across countries. By answering this question, this paper provides a benchmark absent, for instance, political economy frictions or agency problems such as corruptible tax collectors.

I study an environment related to the ones in Murphy et al. (1993) and Acemoglu (1995) in which agents with heterogeneous productivity can produce, appropriate (that is, "grab") others' resources, or work in enforcement. The assumption is that those who produce output in doing so acquire the right to consume it. The First Best outcome sees neither appropriation nor enforcement and property rights are perfectly secure. I assume that the planner can neither force agents into an occupation nor verify the incomes of agents she wants to tax. That is, besides choosing a different occupation than designated by the planner altogether, agents can misrepresent their income in order to be subject to a different tax payment and the planner has no way of checking the true income. However, in order to do so they have to hide some of their resources, which is costly. I show that, even though this technological friction constrains the planner, she always implements an outcome with perfectly secure property rights. She achieves that using both employment of enforcement personnel and income redistribution, which resembles the link between institutional investment and redistribution emphasized by Koeppl et al. (2014) for efficient contract enforcement in a production economy. In fact, the planner may pay subsidies to encourage rather unproductive agents to abstain from appropriation and engage in production, thereby contributing to the pie available to society. In addition, she uses the sector for enforcement to absorb all potential appropriators. Similar to appropriation, employment in enforcement is an unproductive and redistributive activity financed out of the pie to which it does not contribute. However, it does not directly hurt agents' incentives to be productive. Higher costs of hiding (or otherwise misrepresenting) taxable income allow for more income redistribution, leading to a steeper tax schedule. The "marginal" tax payment, the tax paid by the least productive producer, is lower, and possibly a higher subsidy. The planner employs fewer personnel in enforcement and more agents produce more output, increasing aggregate consumption and thus welfare.

These results arise for three reasons. First, I assume that property rights are enforced by personnel whose wellbeing the planner values. Second, besides the first order effect of decreasing the payoffs from production and appropriation, through enforcement, taxation has a second order equilibrium effect on all agents' payoffs. When more taxation maps directly into more enforcement, then, at the margin, production becomes more profitable relative to appropriation. At the same time, working in enforcement becomes more profitable compared to appropriation, too. However, when hiring enforcement personnel, the planner only needs to compensate them for their forgone outside option, which in equilibrium is appropriation and becomes less profitable with taxation and enforcement. In conclusion, it is always beneficial for the planner to tax producers slightly more and hire more enforcement personnel. Doing so absorbs potential appropriators and induces more agents to produce. This effect is only strengthened when the planner can pay subsidies to unproductive producers. Finally, in equilibrium, enforcement uses unproductive rather than productive factors making perfectly secure property rights affordable. Enforcement personnel is recruited from a pool of agents that would otherwise engage in appropriation, not in production.

This paper is motivated by the fact that members of every society can decide to engage either in production or in appropriation activities which are unproductive and purely redistributive. In the context of this paper, appropriation activities are for instance rent-seeking, corruption, outright theft and property crimes, fraud, extortion, and expropriation by government officials. These interpretations seem to be the most relevant ones for developing countries. They also connect the paper to the large related literatures started by Tullock (1967), Rose-Ackerman (1975), and Becker (1968), respectively. For simplicity, I focus on activities that do not target particular groups and do not require any specialized productive ability, as opposed to more skill-intensive activities such as financial fraud. The availability of these activities affects the allocation of talent and resources (for example, Baumol (1990), Murphy et al. (1991, 1993), Acemoglu (1995), Acemoglu and Verdier (1998), Grossman and Kim (1995, 1996)) and implies that (in)secure property rights are an endogenous outcome. These implications in turn affect the expected returns to all sorts of investments and thus many economic decisions, with implications for development and growth; see, for instance, Hall and Jones (1999) and Murphy et al. (1991). Among others, Knack and Keefer (1995), Barro (1996), Easterly and Levine (2003), Acemoglu and Johnson (2005), Acemoglu et al. (2005), and Rodrik et al. (2004) have argued that institutions in a broad sense are an important determinant of economic development and growth. I explore how a particular technological friction in the collection of tax payments required to finance enforcement might affect a society's choice of more or less secure property rights. The literature on taxation and tax evasion suggests that this type of friction is of empirical relevance. Schneider and Enste (2000), for example, provide estimates of "shadow" and underground economic activity suitable to hide income from taxation and show that it is quite prevalent in developing and transition economies but also widespread in developed countries. Similarly, Andreoni et al. (1998) report that tax underpayment and evasion are not uncommon in developed countries and can be fairly widespread in developing countries.

In related work, Skogh and Stuart (1982) show that financing property rights enforcement through taxation can benefit a society; Teng (2000) analyzes the interaction of taxation and property rights enforcement when they exhibit economies of scope and a specific complementarity; and Gonzalez and Neary (2008) study optimal taxation in a growth economy when there is conflict over the distribution of income. None of these works addresses the problem I study. Neither does the literature on optimal income taxation and public good provision without the need to elicit preferences, such as Boadway and Keen (1993) and Brett and Weymark (2008). By contrast to work in this literature, the public good enforcement in this paper is not valued except through the expected payoffs from occupational choices. It affects the payoff structure in the economy and thus economic decisions that affect the tax base. It is more similar in nature to productive public goods as in Barro (1990) and Glomm and Ravikumar (1994), who discuss public infrastructure-type services. Lockwood et al. (2015) study optimal taxation when heterogenous agents have a discrete occupational choice and can generate taxable income from rent-seeking (see also Rothschild and Scheuer (2015)). They abstract from the use of the tax receipts to focus on the allocation of talent. In my model, taxation finances the payroll of one of the available occupations and appropriation income cannot be taxed. Examples of the related literature that studies the effects of costly income falsification are Lacker and Weinberg (1989) on optimal contracts and Grochulski (2007) on optimal taxation. Both focus on alleviating individual income risk. More generally, the focus of my paper is not the optimal design of the tax schedule but the implications of the friction in raising funds on the security of property rights. I therefore abstract from a general description of the costs of hiding income and how its details affect the optimal tax schedule.

In the environment I consider, the friction constrains the planner's allocation, yet appropriation is absent and property rights are secure. However, when the decision maker maximizes an objective function that might arise from frictions in the political arena, she implements an outcome in which appropriation is present and property rights are not perfectly secure. In fact, the example I provide may look a lot like a predatory state (see, for instance, Moselle and Polak (2001)). This result lends further support to the view that political economy frictions, the interplay of socio-economic groups, and the interaction of economic rents with political power shape property rights. Prominent examples of work on limited commitment, rent extraction, and inefficient redistribution are Acemoglu (2003, 2006, 2008) and Acemoglu and Robinson (2001).

Finally, I do not address what determines how high a cost misrepresenting agents face. This cost is an endogenous economic institution and related to the focus of the large literature on optimal income tax enforcement started by Reinganum and Wilde (1985), which connects to the literature on tax evasion started by Allingham and Sandmo (1972). (Also, see for instance Cowell and Gordon (1988) and Falkinger (1991) on the interaction of public good provision and tax evasion.) One can, however, imagine a well developed country with a well governed and equipped tax authority; a well developed financial market in which participants have effective screening devices available and many companies are required to regularly report to and get audited by agents of other participants; with a relatively important heavy and manufacturing industry producing tangible output; and with relatively many big enough companies that require a well established organizational form with cross checks and a within-firm bureaucracy to operate effectively. One can also imagine a not so well developed country with an understaffed tax authority and a large informal sector; with a relatively important agrarian sector; with large rural areas that are far behind the urban centers, both technologically and administratively; in which most firms are small enterprises, with a single owner often being the single employee. One may expect that it would be more difficult and thus costly to hide income and escape reporting duties in the former society. While the exogeneous cost of misrepresenting taxable income precludes a sensible welfare analysis, my results suggest that societies should make misrepresenting more costly to improve their outcome.

2 The Model

Consider a static one period economy with a unit measure of agents, a single consumption good, and a benevolent planner that maximizes aggregate welfare. Preferences are identical across agents and represented by the risk-neutral utility function $u(c) = c$. Agents are heterogeneous with respect to their productivity $w \in [0, 1]$. I assume that w is drawn from a publicly known distribution with cumulative distribution function $F(w)$ on the support $[0, 1]$, differentiable density $f(w) > 0$ on $(0, 1)$, and mean μ. An individual's productivity is, however, her private information and cannot be verified by the planner. I assume that the productivity distribution does not have too much mass on very unproductive agents and that its density is not too elastic: $F(0.1) \leq 0.9$ and $-\frac{wf'(w)}{f(w)} \leq 2$ for all $w \in (0, 1)$. (These assumptions are sufficient but not necessary for the results.) For ease of notation I define $\bar{w}$ by $F(\bar{w}) = 0.9$ and thus assume $\bar{w} \geq 0.1$. Agents are endowed with one unit of time that is supplied indivisibly to an occupation in one of three mutually exclusive sectors. An agent with productivity $w \in [0, 1]$ can decide to either produce w units of the consumption good, work in an enforcement sector for a fixed and certain wage w^e, or engage in appropriation activities. Let Ω^p, Ω^a, and Ω^e be the sets of producers, appropriators, and enforcers with measures ω^p, ω^a, and ω^e, respectively.

After production, producers display an income z that may or may note equal their actual income w (i.e., the output of their productive activity) to the planner and pay a tax (or possibly receive a subsidy) $t(z) \in [-1, 1]$ that is indexed by and may vary with the income they display. Following Lacker and Weinberg (1989), this assumptions is without loss of generality (see appendix A). For notational simplicity, I use the shorthand $t_z = t(z)$. When displaying an income z different from actual income w, agents have to hide the resources they do not display–at a cost $\phi \in [0, 1]$ per unit of income hidden. They cannot display more resources than they actually have. I assume that it is impossible to hide all output produced. The income display $z(w)$ of an agent with income w thus has to be in the set $(0, w]$. Therefore, after displaying income $z(w)$ at the cost $\phi(w - z(w))$ and paying taxes $t_{z(w)}$, a producer with productivity w carries resources $w - t_{z(w)} - \phi(w - z(w))$.

After production and tax payments but before consumption, there are two rounds of random matching between agents. In the first round, every agent can meet either an appropriator, a producer, or an enforcer and appropriation takes place. In the second round, successful appropriators, while running off, are randomly matched with (i.e., run into) another agent and may be apprehended if they happen to run into an enforcer. Enforcement personnel is clearly recognizable, cannot be expropriated of their resources (they enforce their own property rights perfectly), and cannot apprehend agents that did not violate anybody else's property rights. I assume that the probability p of any agent meeting an appropriator equals the measure of appropriators, ω^a. This specification is similar to the one İmrohoroğlu et al. (2000) use. Similarly, the probability q of any agent meeting a producer equals the measure of producers, ω^p, and the probability $(1 - \theta)$ of any agent meeting an enforcer equals the measure of enforcers, ω^e. If a producer with productivity w is matched with an appropriator in the first round, she loses all her resources $w - t_{z(w)} - \phi(w - z(w))$ to the appropriator. That is, she cannot hide her income from an appropriator she is matched with in the same way as she can hide it from the planner because appropriators do not face institutional or resource constraints on finding and grabbing income, once they prey on somebody. If she meets another producer or an enforcer, then they just chat and walk off. If an appropriator meets a producer with productivity w, she runs off with the resources $w - t_{z(w)} - \phi(w - z(w))$ the producer carries. If she meets another appropriator, then there is nothing to appropriate and both walk off empty-handed. If she meets an enforcer, then she recognizes that and realizes that there is again nothing to appropriate and so she does not try; the enforcer cannot apprehend a potential offender that did not get to commit an offense, so, both walk off. If enforcers meet they chat and walk off.

In the second round, successful appropriators have their hands full carrying resources and cannot appropriate anything more if they were to run into another producer or another successful appropriator carrying resources. Unsuccessful appropriators cannot expropriate successful ap-

Table 1

Timing in the Underlying Economy

1. Production	2. Enforcement Provision	3. Meeting 1	4. Meeting 2	5. Consumption
Producers produce. Others do not.	Producers pay taxes, enforcement is implemented.	Appropriation takes place.	Apprehension takes place.	Consumption takes place.

propriators on the run. Producers, expropriated or not, cannot apprehend an appropriator on the run. However, if a successful appropriator meets an enforcer, the appropriator loses all the resources she carries and the enforcer returns them to the producer who was expropriated of them. That is, the probability of apprehension equals the measure of enforcement personnel in the population. The apprehended appropriator does not incur any additional costs besides zero consumption. Finally, after the second round, all agents consume the resources they have in hand.

In the beginning of the period, the planner instructs all agents what their occupation is. She then collects taxes from producers, and pays enforcement personnel. She chooses the instructions, the tax schedule, the measure of enforcement personnel, and the wage to be paid to them in order to maximize the sum of all agents' expected payoffs. Her choice of what measure of enforcement personnel to employ amounts to choosing (a level of investment into) the apprehension technology, which determines the probability of apprehension (see, for instance, İmrohoroğlu et al. (2000)). She has to maintain a balanced budget, that is, the expenses for enforcement personnel have to equal the taxes collected: $\omega^e w^e = \int_{w \in \Omega^p} t_{z(w)} f(w) dw$. While budget balance would arise anyway, imposing it simplifies the analysis.

The timing in the economy is summarized in Tables 1 and 2. Table 1 depicts the timing in the underlying economy as described above. Table 2 gives the larger picture. In the beginning of the period, the planner chooses and implements a regime of taxes and enforcement and announces her instructions for all agents. Thereafter, given the prevailing regime and the planner's instructions, agents engage in activities of their choice. Producers produce and pay taxes. Then, agents are randomly matched and interact with each other. At this point, appropriators try to appropriate resources from producers. Finally, agents consume.

In Appendix D, I lay out an example economy with a uniform productivity distribution. I use it to illustrate some results. I discuss my modeling choices in section 4.

Table 2

Timeline within the Period

1. Regime Choice	2. Occupations	3. Meetings and Interaction
The planner chooses and implements a regime and instructs all agents.	Given the regime, agents (follow instructions and) choose occupations.	Agents are randomly matched and interact. Then, they consume.

3 Analysis

In this environment, it is beneficial to pay taxes to finance at least some enforcement as it increases producers' expected payoffs through two channels: How likely is it an offender is apprehended and the appropriated resources are returned and, as this likelihood affects the expected payoffs associated with occupational choices, how likely is it to fall victim to appropriation. I start by specifying the actions agents can take and the payoff functions these map into as well as the planner's objective function. After that I state and discuss the implications of the model. I take the economic fundamentals summarized by the distribution function F for productivities and the technology characterized by the cost parameter ϕ as given. All proofs can be found in Appendix E.

3.1 Payoffs and the Planner's Objective Function

Let $\Sigma \equiv [0,1] \times [-1,1]^{[0,1]} \times [0,1] \times \{0,1\}^{[0,1]} \times \{0,1\}^{[0,1]}$. Consider any regime $\sigma \equiv (\theta, t, w^e, \chi^p, \chi^e) \in \Sigma$ where $\theta \in [0,1]$ is the probability of escaping apprehension, w^e is the wage paid in the enforcement sector, t is a function $t : [0,1] \to [-1,1]$ that represents the tax schedule, and χ^p and χ^e are functions $\chi^p : [0,1] \to \{0,1\}$ and $\chi^e : [0,1] \to \{0,1\}$, where $\chi^p(w) + \chi^e(w) \leq 1$ for all w. Taken together, χ^p and χ^e indicate the occupation an agent chooses (or is instructed) to engage in. As with the tax schedule, for notational simplicity, I use the shorthand $\chi^p_w = \chi^p(w)$ and $\chi^e_w = \chi^e(w)$. The planner takes into account the map from her occupational instructions, or agents' occupational choices, to the probabilities p, q, and $(1-\theta)$ of meeting an appropriator, a producer, or an enforcer, respectively, and understands that $q = \omega^p$, $(1-\theta) = \omega^e$, and $p = \omega^a$, where

$$\omega^p = \int_0^1 \chi^p_w f(w)dw; \quad \omega^e = \int_0^1 (1-\chi^p_w)\chi^e_w f(w)dw; \quad \omega^a = \int_0^1 (1-\chi^p_w)(1-\chi^e_w) f(w)dw.$$

Furthermore, $f(w|w \in \Omega^p) = f(w)(\omega^p)^{-1}$ for all $w \in \Omega^p$ and 0 otherwise, $\Omega^p = \{w \in [0,1] : \chi^p_w = 1\}$, $\Omega^e = \{w \in [0,1] : \chi^p_w = 0 \ \wedge \ \chi^e = 1\}$, and $\Omega^a = \{w \in [0,1] : \chi^p_w = 0 \ \wedge \ \chi^e = 0\}$.

Now, a producer meets an appropriator with probability p, in which case she is expropriated of all her resources. With probability $1-\theta$ an enforcer apprehends the appropriator and returns the producer's resources to him or her; with probability θ, the appropriator can run off with them. Therefore, an agent w's expected payoff from production, when displaying income z and paying both the designated tax t_z for producers displaying z and the associated cost of hiding income if $z < w$, is given by

$$\begin{aligned}&(1-\theta)pu[w-t_z-\phi(w-z)]+\theta pu[0]+(1-p)u[w-t_z-\phi(w-z)]\\ =\ &[1-\theta p](w-t_z-\phi(w-z)).\end{aligned}$$

When a producer chooses an income to display, she maximizes her payoff from doing so. However, the income display must be feasible and, given that the planner understands the incentives to falsify income, consistent with all agents' occupational and falsification choices. First, no producer w can display a higher income than they have generated, $z \leq w$. Second, if a producer were to display an income that, given the regime, no agent that produces generates, then the planner catches that and punishes the producer prohibitively high. Third, if the income displayed is one that, given the regime, no producer that generates it would optimally display, then the planner concludes to be facing an attempt to falsify income and punishes the producer prohibitively high. Thus, given any regime $\sigma \in \Sigma$, and in particular a tax schedule t, define producer w's income display $\zeta(w;\sigma)$ to be one that satisfies

$$\zeta(w;\sigma) \in \arg\max_{z\in[0,w]\cap Z(\Omega^p)} \quad w-t_z-\phi(w-z), \tag{1}$$

where

$$Z(\Omega^p)=\{w\in\Omega^p:\zeta(w;\sigma)=w\}$$

is the set of all income displays that are the true income optimally displayed by some producer under the regime σ. In the main text, to simplify notation, I suppress the dependence on the regime σ and write ζ_w to mean $\zeta(w;\sigma)$ whenever there is no risk of confusion. In general, ζ_w does not need to exist. However, starting from the guess that, given a regime $\sigma \in \Sigma$, $\zeta(w;\sigma)$ does exist for all $w \in \Omega^p$, the planner's optimal regime implements a continuous tax schedule and a compact set $Z(\Omega^p)$ which intersects another compact set $[0,w]$, thus verifying the guess. Agent w's expected payoff from being a producer is given by the function $\tilde{\varphi}:\Sigma\times[0,1]\to[0,1]$,

$$\tilde{\varphi}(\sigma;w)=(1-\theta p)(w-t_{\zeta_w}-\phi(w-\zeta_w)).$$

Given σ, conditional on being matched with a producer, which occurs with probability q, an appropriator can expect to run off with resources that, if not apprehended, give her expected utility according to the function $v : \Sigma \to [0,1]$ given by

$$v(\sigma) = \int_{w \in \Omega^p} (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w | w \in \Omega^p) dw.$$

Given Ω^p, $v(\sigma)$ is the same for all appropriators as it is independent of the appropriator's productivity. When running off with the resources appropriated, she meets an enforcer and is apprehended with probability $1-\theta$, in which case she consumes nothing; she gets away with probability θ. Therefore, agent w's expected payoff from being an appropriator is given by

$$\theta q v(\sigma) + (1-\theta) q u[0] + (1-q) u[0] = \theta q v(\sigma).$$

That is, an appropriator who is matched with a producer gets a payoff proportional to a draw from the set of productivities of producers net of taxes and costs of hiding income. An appropriator's expected payoff can be written as a function $\nu : \Sigma \times [0,1] \to [0,1]$ given by

$$\begin{aligned}\nu(\sigma; w) &= \theta q \int_{w \in \Omega^p} (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w | w \in \Omega^p) dw \\ &= \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw,\end{aligned}$$

as $q = \omega^p$ and $f(w | w \in \Omega^p) = f(w)(\omega^p)^{-1}$ for all $w \in \Omega^p$. An agent with productivity w that works in the enforcement sector receives a wage w^e with certainty and her payoff from working in enforcement is thus $u[w^e] = w^e$. The planner's balanced budget can be written as

$$(1-\theta) w^e = \int_0^1 \chi_w^p t_{\zeta_w} f(w) dw.$$

The planner's objective function is given by (see Appendix B)

$$\begin{aligned}&\int_0^1 \left[\chi_w^p \tilde{\varphi}(\sigma; w) + (1-\chi_w^p)\chi_w^e w^e + (1-\chi_w^p)(1-\chi_w^e)\nu(\sigma; w)\right] f(w) dw \\ = &\int_0^1 \chi_w^p (w - \phi(w - \zeta_w)) f(w) dw.\end{aligned}$$

The planner's problem thus is to choose instructions on the occupations to take up, a tax schedule, a wage in enforcement, and a measure of enforcement personnel employed to solve

$$\text{(PP)} \qquad \begin{aligned}&\max_{\sigma \in \Sigma} \int_0^1 \chi_w^p (w - \phi(w - \zeta_w)) f(w) dw \\ s.t. \quad &(1-\theta) w^e = \int_0^1 \chi_w^p t_{\zeta_w} f(w) dw \geq 0; \\ &\omega^p + \omega^e + \omega^a = 1; \quad \omega^p, \omega^e, \omega^a \geq 0; \quad q = \omega^p, \quad (1-\theta) = \omega^e, \quad p = \omega^a.\end{aligned}$$

The constraints the planner always has to observe are the balanced budget and that the probabilities of meeting agents in a specific occupation equal the measures of agents in these respective occupations, which have to be non-negative and add up to one. The non-negative budget basically captures feasibility of the regime. Looking at problem (PP) it is evident that I can focus on instructions that are incentive compatible without loss of generality. Agents can deviate from the planner's prescription σ by taking up a different occupation. If σ' is the equilibrium induced by σ, which differs from σ only by the occupations that a nontrivial set of agents has taken up instead of the ones prescribed by the planner, then it is incentive compatible as there would be profitable deviations otherwise. Then, the incentive compatible regime σ' induces the same equilibrium allocation and payoffs as σ. I summarize this observation in the following statement.

Observation 1. *For any regime $\sigma \in \Sigma$, there is a regime $\sigma' \in \Sigma$ with incentive compatible instructions that induces the same equilibrium allocation and payoffs as σ.*

I first study the efficient outcome if the planner is unconstrained and can dictate occupational choices. Then, I analyze the case of a constrained planner that cannot control agents' occupational choices. Thereafter, I analyze the case in which incomes are unverifiable and producers can incur a cost to display a different income so as to be subject to a different tax payment.

3.2 The First Best Outcome

In this section, I take a step back and describe the first best outcome. Suppose the planner is absolutely unconstrained so that she can not only choose a tax schedule, the employment in enforcement, and the wage paid in the enforcement sector but also instruct agents what occupation to take up, irrespective of whether or not they find it optimal to do so, and verify producers' taxable incomes. That is, the tax schedule is contingent on the producers' actual income w rather than an income displayed and there are no costs of hiding income. The planner's problem then is

$$\text{(FBP)} \qquad \max_{\sigma \in \Sigma} \int_0^1 \chi_w^p w f(w) dw$$
$$s.t. \quad (1-\theta) w^e = \int_0^1 \chi_w^p t_w f(w) dw \geq 0,$$
$$\omega^p + \omega^e + \omega^a = 1; \quad \omega^p, \omega^e, \omega^a \geq 0; \quad q = \omega^p, \quad (1-\theta) = \omega^e, \quad p = \omega^a.$$

The following proposition characterises the first best outcome.

Proposition 1 (First Best Outcome)**.** *The efficient outcome is that all agents produce and no enforcement is implemented at all. The planner imposes $\chi_w^p = 1$ for a.e. $w \in [0,1]$, $\theta = 1$, and $\int_0^1 t_w f(w) dw = 0$. The objective function attains the value μ.*

This First Best outcome derives from two aspects. First, both appropriation and enforcement are purely redistributive. Any agent that appropriates (or enforces) does not produce and thus not contribute to the pie the planner has available for distribution. Second, linear utility implies that the unconstrained planner does not have an a priori incentive to redistribute resources among agents through transfers. As a consequence, the planner just maximizes output so that everybody produces and any tax schedule that only redistributes incomes through tax payments and subsidies is efficient. In particular, collecting no taxes (and thus not paying any transfers) at all is an optimum. Notice that, while enforcement is costly due to foregone production by personnel employed in enforcement, the resources spent on it are consumed by enforcement personnel and thus enter the planner's objective.

Now assume that the planner cannot control individual occupational choices. She can verify incomes and choose the regime consisting of employment in enforcement, the wage paid in enforcement, and the tax schedule, which still is contingent on income w. Without loss of generality, I focus on instructions that are incentive compatible. Agents can deviate from the planner's prescription σ by taking up a different occupation. If σ' is the equilibrium induced by σ, which differs from σ in the occupations a non-trivial set of agents has taken up, then it is incentive compatible as there would be profitable deviations otherwise. Then, the incentive compatible regime σ' induces the same equilibrium allocation and payoffs as σ. The planner's problem is

$$\text{(FBP}'\text{)} \qquad \max_{\sigma\in\Sigma} \int_0^1 \chi_w^p w f(w) dw$$

$$\begin{aligned}
s.t.\quad & (1-\theta)w^e = \int_0^1 \chi_w^p t_w f(w)dw \geq 0, \\
& (1-\theta p)(w-t_w) \geq \max\left\{w^e; \theta \int_0^1 \chi_w^p (w-t_w) f(w)dw\right\} \quad \text{for a.e. } w \in \Omega^p, \\
& w^e \geq \max\left\{(1-\theta p)(w-t_w); \theta \int_0^1 \chi_w^p (w-t_w) f(w)dw\right\} \quad \text{for a.e. } w \in \Omega^e, \\
& \theta \int_0^1 \chi_w^p (w-t_w) f(w)dw \geq \max\left\{(1-\theta p)(w-t_w); w^e\right\} \quad \text{for a.e. } w \in \Omega^a, \\
& \omega^p + \omega^e + \omega^a = 1; \quad \omega^p, \omega^e, \omega^a \geq 0; \quad q = \omega^p, \quad (1-\theta) = \omega^e, \quad p = \omega^a.
\end{aligned}$$

The three additional constraints capture the incentive problem the planner faces. Each agent should expect a payoff from the occupation she is instructed to take up that is at least as high as the maximum payoff she could obtain from either one of the alternative occupations. The following result states that the solution to problem (FBP′) implements the First Best outcome.

Proposition 2 (Incentive Compatible Instructions)**.** *The planner instructs all agents to produce and no enforcement is implemented at all. She uses taxes to redistribute completely. The* *unique solution is* $\chi_w^p = 1$ *and* $t_w = w - \mu$ *for a.e.* $w \in [0,1]$, $\int_0^1 t_w f(w)dw = 0$, *and* $\theta = 1$. *The objective function attains the value* μ.

The planner whose occupation assignments have to be individually optimal given the regime chosen instructs all agent's to produce so as to maximize the pie. She then collects taxes and subsidizes unproductive agents with transfers paid for by productive agents. The transfers equalize consumption across agents so as to incentivize abstention from appropriation. No resources are spent on enforcement at all. The intuition is exactly the same as before. Appropriation and enforcement (and enforcement personnel) do not contribute to the pie. Hence, as long as subsidies to potential appropriators can make the efficient outcome incentive compatible, there will be no enforcement and everybody produces so as to maximize the pie available for consumption in society. This result is very general: see Appendix C for a brief discussion of more general concave utility functions.

3.3 The Constrained Planner

In this section, I allow producers to misrepresent their income by a false display in order to be subject to a different tax payment. The planner cannot verify the income an agent displays. In order to display a false income, agents have to incur a cost $\phi \in [0,1]$ per unit of resources they are hiding from taxation. Consider problem (FBP′) when agents can display false income.

$$\text{(PP}'\text{)} \quad \max_{\sigma \in \Sigma} \int_0^1 \chi_w^p (w - \phi(w - \zeta_w)) f(w) dw$$

s.t.,

$$\text{(2)} \quad (1-\theta) w^e = \int_0^1 \chi_w^p t_{\zeta_w} f(w) dw \geq 0;$$

for a.e. $w \in \Omega^p$,

$$\text{(3)} \quad (1-\theta p)(w - t_{\zeta_w} - \phi(w-\zeta_w)) \geq \max\left\{ w^e ; \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw \right\};$$

for a.e. $w \in \Omega^e$,

$$\text{(4)} \quad w^e \geq \max\left\{ (1-\theta p)(w - t_{\zeta_w} - \phi(w-\zeta_w)) ; \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw \right\};$$

for a.e. $w \in \Omega^a$,

$$\text{(5)} \quad \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw \geq \max\left\{ (1-\theta p)(w - t_{\zeta_w} - \phi(w-\zeta_w)) ; w^e \right\};$$

$$\text{(6)} \quad \omega^p + \omega^e + \omega^a = 1; \quad \omega^p, \omega^e, \omega^a \geq 0;$$

$$\text{(7)} \quad q = \omega^p, \quad (1-\theta) = \omega^e, \quad p = \omega^a.$$

Problem (PP′) captures the focus on occupation instructions that are incentive compatible when producers may display false income. Constraint (2) captures the budget balance requirement and constraint (6) requires consistent occupational assignments in the sense that the measures of agents in each occupation have to be positive and add up to one. Constraint (3) says that a producer that may choose to display a false income should be at least as well off as she would be taking up an activity in enforcement or appropriation. Similarly, constraints (4) and (5) require agents that are assigned to be enforcers or appropriators to find it optimal to follow the occupational instructions instead of deviating to production and hiding income. Constraint (7) states that the planner understands and takes into account how the occupation assignments map into the probabilities that enter the expected payoffs.

As is intuitive, the planner chooses to have the most productive agents produce output.

Lemma 1. *In any solution to problem* (PP′)*, there exists a* $w^* \in [0, 1)$ *such that* $\Omega^p = [w^*, 1]$*.*

Using lemma 1, one can show that the planner avoids tax schedules that induce a nontrivial set of agents to hide income, because it generates deadweight loss.

Proposition 3. *Any solution to problem* (PP′) *precludes income hidden from taxation.*

This result implies that we can focus on allocations that preclude any producers to have an incentive to hide income from taxation. Lemma 1 and proposition 3 in hand, I can simplify the problem by restricting attention to a subset of the original constraint set. Moreover, I can adopt a more convenient notation that appreciates the fact that all agents at least as productive as w^* produce. Note that the cutoff w^* implies that $\chi^p_w = 1$ for all $w \geq w^*$ and zero otherwise. It follows that $\omega^p = (1 - F(w^*))$ and, as both $\omega^e \geq 0$ and $\omega^a \geq 0$, $\omega^e = (1 - \theta) \leq F(w^*)$. Let $\Sigma' \equiv [0, 1] \times [-1, 1]^{[0,1]} \times [0, 1] \times [0, 1]$ with a generic element

$\sigma' = (\theta, t, w^e, w^*) \in \Sigma'$. The planner's problem then is

$$
\begin{aligned}
&(\mathrm{PP}'') && \max_{\sigma' \in \Sigma'} \int_{w^*}^{1} w f(w) dw \\
& && s.t., \\
&(\mathrm{bbc}) && (1-\theta) w^e = \int_{w^*}^{1} t_w f(w) dw \geq 0; \\
&(\mathrm{p1}) && (1-\theta p)(w - t_w) \geq \max\left\{ w^e, \theta \int_{w^*}^{1} (w - t_w) f(w) dw \right\} \quad \text{for a.e. } w \in \Omega^p; \\
&(\mathrm{p2}) && (1-\theta p)(w - t_w) \geq (1-\theta p)(w - t_{w'} - \phi(w - w')) \quad \text{for a.e. } w, w' \in \Omega^p,\ w > w'; \\
&(\mathrm{e1}) && w^e \geq \max\left\{ (1-\theta p)(w - t_w), \theta \int_{w^*}^{1} (w - t_w) f(w) dw \right\} \quad \text{for a.e. } w \in \Omega^e; \\
&(\mathrm{a1}) && \theta \int_{w^*}^{1} (w - t_w) f(w) dw \geq \max\left\{ (1-\theta p)(w - t_w), w^e \right\} \quad \text{for a.e. } w \in \Omega^a; \\
&(\mathrm{coc}) && \omega^p + \omega^e + \omega^a = 1; \quad \omega^p, \omega^e, \omega^a \geq 0; \\
&(\mathrm{prob}) && q = \omega^p, \quad (1-\theta) = \omega^e, \quad p = \omega^a.
\end{aligned}
$$

This problem (PP$''$) is the very problem (PP), constrained to a subset of the original constraint set, and the notation adjusted to that subset. In particular, in that subset, income displays are truthful so that $\zeta(w; \sigma) = w$ for all producers w given the regime σ. Observation 1, lemma 1, and proposition 3 establish that any candidate solution to problem (PP) has to be in the constraint set of problem (PP$''$) as it would otherwise be dominated by some element of the latter. If problem (PP$''$) has a solution, then it dominates all other elements of the constraint set of (PP$''$). At the same time, each element of the constraint set of (PP) is dominated by some element of the constraint set of (PP$''$). It is thus sufficient to establish the existence of and characterize the solution to problem (PP$''$) to solve problem (PP).

Constraints (p1), (e1), and (a1) state that an agent in the respective occupation should expect at least as high a payoff as she could expect from either one of the other two occupations. Constraint (p2) states that no producer w should be willing to deviate from the planner's instructions and produce, display an income equal to some $w' < w$, and incur both the associated tax payment $t_{w'}$ and the cost from hiding output $(w - w')$. Both appropriators and enforcers have productivity $w < w^*$ and cannot pretend to have produced more than that were they to deviate to production. Therefore, a constraint capturing their incentive for a double deviation to production followed by a false income display is not needed. Constraint (p2) implies that, whenever $\theta p < 1$, for all $w, w' \in \Omega^p$, $w > w'$,

$$
\phi w - t_w \geq \phi w' - t_{w'}. \tag{8}
$$

If $\phi = 1$, the planner can implement the First Best outcome with a small enough announced wage in enforcement, e.g., $w^e = 0$. I thus focus on the interesting case when $\phi \in [0, 1)$.

Constraint (bbc) is the budget balance constraint. Constraint (coc) requires the occupational choices to be consistent so that the measures of agents in all occupations are nonnegative and add up to one. Finally, the equalities collected in constraint (prob) state that the planner understands and takes into account how the occupation assignments she makes map into the probabilities that enter the expected payoffs. The first interesting point is that the solution of the constrained planner's problem implements enforcement.

Lemma 2. *In any solution to problem (PP″), there is a positive measure of enforcers.*

Compared to the first best outcome, the constrained planner incurs a cost of hiring personnel to enforce property rights. The following result characterizes the solution to problem (PP″).

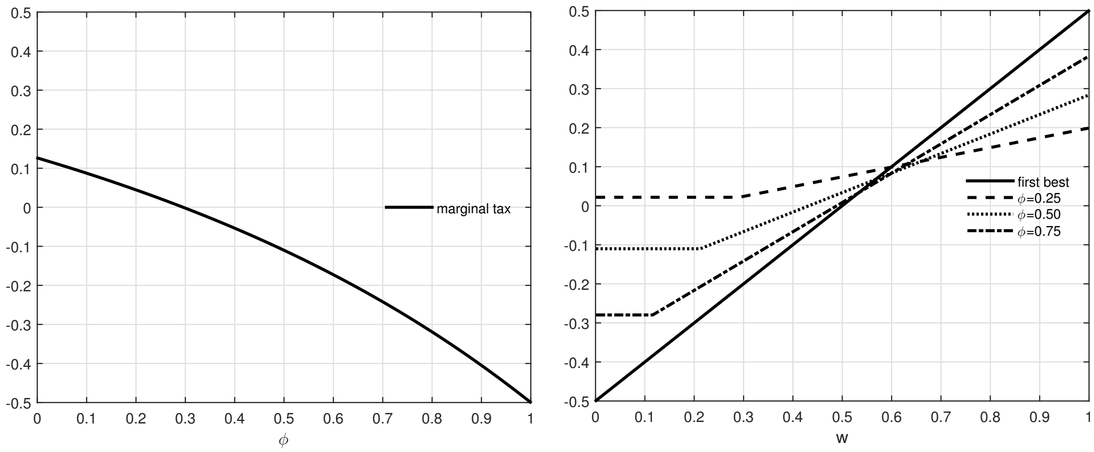
Proposition 4 (Unverifiable Incomes)**.** *Given a cost $\phi \in [0, 1)$ of hiding resources, the planner's problem has a unique solution. There is a productivity cutoff $w^* \in (0, 1)$ such that all agents with productivity of at least w^* produce and all agents with productivity less than w^* are employed in enforcement, except for possibly a measure zero set of agents that engage in appropriation. The planner implements a tax schedule $t_w = (t_{w^*} - \phi w^*) + \phi w$ for all $w \geq w^*$, a linear and strictly increasing function of income w if $\phi > 0$ and constant otherwise. A society with higher costs of hiding income implements a steeper tax profile with a smaller minimum tax, employs fewer enforcement personnel at a higher wage, sees more productive activity, produces more output, and experiences higher welfare.*

This result says a number of things. The planner chooses to employ enough enforcement personnel so as to crowd out appropriation altogether and property rights are perfectly secure. She does so not by deterrence or punishment but by absorbing potential appropriators into the sector for enforcement. That is, unproductive agents are employed in enforcement at a fixed wage. Appropriation will never be attempted so that enforcement personnel does not actually provide a service other than being present (and thus effectively deterring appropriation efforts). Being employed in enforcement amounts to refraining from appropriation. The wage paid in enforcement amounts to a transfer financed by productive agents. Employment in enforcement thus institutionalizes redistribution that would otherwise take place through expropriation. If the costs of hiding output from taxation are higher, then the planner can implement more redistribution through taxation. She can and does then choose to employ fewer agents in enforcement, implement a steeper tax schedule, and possibly subsidize production by unproductive producers.

While taxes may distort the occupational choice between production and appropriation, higher tax payments induce a first order decrease in the expected payoffs from both production and appropriation. This effect is present in general, independent of the exact tax schedule in place and as long as utility is increasing. So, tax payments affect the choice between either one of those two occupations and employment in enforcement. Too high taxes may draw productive agents (as well as appropriators) into the enforcement sector. The threat of expropriation of the resources an agent carries around, however, affects the occupational choice between production and appropriation; as does the probability of getting apprehended, albeit in the opposite direction. Intermediated by the sector for enforcement, tax payments increase the probability of productive agents being able to reap the returns to their productive activity, which increases the incentive to produce. At the same time, the probability of appropriation efforts being successful decreases, which decreases the incentive to engage in appropriation activities. At the margin, this second order effect induces more agents to prefer to produce. It, too, is present in general, independent of the exact tax schedule in place and as long as utility is increasing. Inframarginally, unproductive agents lean more towards switching to an occupation in the enforcement sector. Thus, the planner optimally employs rather unproductive members of society as enforcement personnel at a wage that makes them indifferent between enforcement and appropriation. As she absorbs all potential appropriators into the enforcement sector, there is no appropriation and property rights are perfectly secure. This security of property rights derives from the availability of a sector for enforcement of those rights because it provides an alternative occupation. That occupation is unproductive, as is appropriation. But, in contrast to appropriation, it does not harm the incentives of more productive agents to actually produce. Perfectly secure property rights are not too costly because enforcement personnel is recruited from a pool of agents that would otherwise engage in appropriation. While enforcers do not produce, enforcement does not withdraw agents from productive activities. Moreover, the resources spent on enforcement finance the consumption of enforcement personnel, which the planner values.

The tax profile that finances enforcement is redistributive. High income agents pay higher taxes than–and may even finance transfers to–low income agents. In fact, the “marginal” tax, the tax paid by the least productive producer, may well be negative if the cost of hiding income is high enough. Negative taxes for unskilled producers subsidize their production, which contributes to the pie society has available for distribution amongst its members. These subsidies also incentivize abstention from appropriation, which would hurt others’ incentives to produce. As an example, Figure 1(a) depicts the marginal tax as a function of the cost of misrepresenting for an economy in which the productivity distribution is uniform. It is negative for (not particularly) high values of ϕ.

In societies in which it is more costly for agents to hide income from taxation it is easier to implement more redistributive tax profiles. That is, the tax profile can be steeper and the marginal tax can be lower–and subsidies can be higher. Figure 1(b) depicts the tax profiles for a few selected values of the cost parameter ϕ and compares them to the First Best tax



(a) The marginal tax payment t_{w^*}, the tax paid by the marginal producer, decreases with the cost of hiding income from taxation. It is frequently a subsidy to incentivize production.

(b) Example tax schedules and the First Best: With increasing costs of hiding income from taxation, the tax schedule approaches the First Best tax schedule, implementing more redistribution.

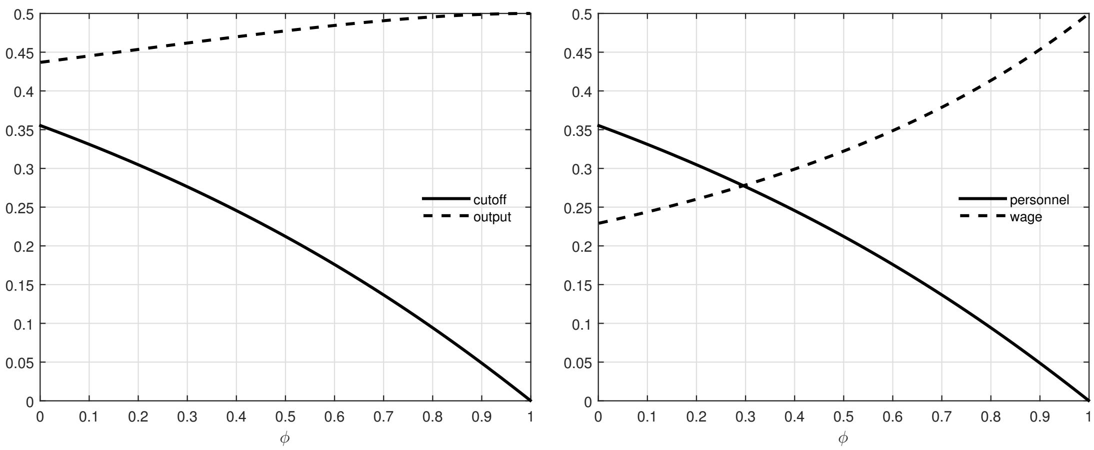
Figure 1

The Tax System

schedule. The flat part at low productivities is due to the assumption that all agents that do not produce were to face the marginal tax if they produced. It however helps to visualize the productivity cutoff and thus where the tax profile starts effectively. All tax profiles are increasing and higher costs of misrepresenting imply steeper profiles with more tax payers (or subsidy receivers), i.e., more productive agents, and a lower marginal tax payment. None of them is as steep as the First Best profile. The constraint implied by the friction in tax collection is binding and effective.

Such a steeper tax profile encourages some agents to produce that would try to appropriate or work in enforcement if the tax profile were to implement less income redistribution. As a consequence, fewer agents work in enforcement and more agents produce. Thus, more output is produced and available to share among the members of society, which increases welfare. Since aggregate consumption increases, agents employed in enforcement get their share of that increase so that their wage is higher. Figure 2(a) depicts the productivity cutoff and the economy's output (which also is the measure of aggregate consumption and welfare) as functions of the cost of misrepresenting. Figure 2(b) does the same for the measure of enforcement personnel employed and the wage they are paid.

The friction in the tax collection technology induces a binding and effective constraint on the planner's decision. Yet, the planner implements an outcome with perfectly secure property rights and no appropriation ever attempted. This result hints at the need for frictions in the



(a) The productivity cutoff decreases and output and thus aggregate consumption increase with the cost of hiding income from taxation. More redistributive taxation incentivizes production that contributes to the pie available to society.

(b) With increasing costs of hiding income from taxation, more redistribution takes place through taxation instead of employment in enforcement. More output and aggregate consumption gives fewer enforcement personnel a higher wage.

Figure 2

Economic Activity and the Enforcement Sector

political arena affecting the decision maker's objective function to generate insecure rights to property.

3.4 A Political Economy Friction

In this section, I ignore the technological friction studied so far. Instead, I provide an example of an optimization problem that could be arising from a friction in the political arena. The decision maker's objective function differs from the one the planner maximizes. The resulting solution to the optimization problem allows for appropriation to take place. A very simple example would be to consider a situation in which the political class favors appropriators. One can think of a corrupt bureaucracy that favors corrupt bureaucrats. Clearly, the implemented regime will allow for appropriation to occur and be profitable in equilibrium. Since this case is not very interesting, I focus on another one. Suppose that the decision maker can reap a rent from her position. She can collect taxes, use some of the receipts to pay for enforcement personnel, and keep the rest. She is basically a dictator. For this economy to allow for a meaningful outcome, I assume that producers can "go informal" and retain a fixed amount $\alpha \in (0, \mu)$ of resources, independent of their productivity. These retained resources can, however, be appropriated by other agents. Another interpretation is that agents' incomes are verifiable but that they can choose to hide all their income from taxation, in which case they retain only α. That is, if the decision maker asks too high tax payments from an agent she might choose to hide all her output. I define $\bar{\alpha}$ as

$$\bar{\alpha} \equiv \frac{1}{(1-F(\bar{w}))^{-1} + F(\bar{w})} \int_{\bar{w}}^{1} w f(w) dw. \tag{9}$$

For this section, I then maintain the following assumption.

Assumption 1. *The resources $\alpha \in (0, \mu)$ retained under informality satisfy $\alpha \leq \bar{\alpha}$.*

Since $\alpha < \mu$, the additional constraint is not binding in problems (FBP) and (FBP′), so that adding it does not alter the First Best solution. The assumption captured in $\alpha \leq \bar{\alpha}$ assures that the equilibrium wage in enforcement satisfies $w^{e*} \geq \alpha$ for all $\phi \in [0, 1)$, implying that adding this additional constraint does not alter the solution to problem (PP). The tax schedule the planner implements satisfies $w - t_w > w^* - t_{w^*} \geq \alpha$ for all $w > w^*$ and $\phi \in [0, 1)$.

Now, consider the optimization problem

$$\begin{aligned}
\text{(DP)} \quad & \max_{\sigma \in \Sigma} \int_0^1 \chi_w^p t_w f(w) dw - (1-\theta) w^e \\
s.t. \quad & (1-\theta p)(w - t_w) \geq \max \left\{ (1-\theta p)\alpha; w^e; \theta \int_0^1 \chi_w^p (w - t_w) f(w) dw \right\} \quad \text{for a.e. } w \in \Omega^p, \\
& w^e \geq \max \left\{ (1-\theta p)\alpha; (1-\theta p)(w - t_w); \theta \int_0^1 \chi_w^p (w - t_w) f(w) dw \right\} \quad \text{for a.e. } w \in \Omega^e, \\
& \theta \int_0^1 \chi_w^p (w - t_w) f(w) dw \geq \max \left\{ (1-\theta p)\alpha; (1-\theta p)(w - t_w); w^e \right\} \quad \text{for a.e. } w \in \Omega^a, \\
& (1-\theta) w^e \geq 0, \ \omega^p + \omega^e + \omega^a = 1, \ \omega^p, \omega^e, \omega^a \geq 0, \ q = \omega^p, \ (1-\theta) = \omega^e, \ p = \omega^a.
\end{aligned}$$

The objective is to maximize the remaining amount of the tax receipts minus the cost incurred by paying for employment in enforcement. Compared to problem (FBP′), the decision maker ignores the budget balance constraint; but the costs of enforcement cannot be negative. The other constraints are the same as in problem (FBP′) except for the additional option of producing, hiding all the output, and retaining only α. They require that (almost) all agents are at least as well off in the occupation they have been assigned as they would be in any other occupation. Additionally, a producer's after tax income should be at least as high as α and enforcers and appropriators should not prefer to produce and retain α. The following holds.

Proposition 5 (Political Economy Friction)**.** *The decision maker chooses a regime so that all agents with productivity $w \geq \alpha$ produce and pay taxes $t_w = w - \alpha$. She does not hire any enforcement personnel so that $\omega^e = 0$ and there is a measure $\omega^a = F(\alpha) > 0$ of appropriators.*

The decision maker taxes producers as much as she can to extract as many resources as possible. When α is small, this outcome looks like a predatory state (Moselle and Polak

(2001)). She does not implement any enforcement and thus does not incur any costs. Most importantly, she does allow for a positive measure of agents to engage in appropriation. The reason is that enforcement is costly. She prefers to consume the resources extracted from producers rather than spending them on enforcement. This result indicates that, since appropriation is feasible, one can certainly find an objective function that induces a society's decision making process to deliver outcomes with imperfectly secure property rights. Frictions in the political arena are major candidates for playing a leading role in that respect.

4 Discussion

In this section, I briefly discuss the modeling choices that seem to require justification.

The planner's objective. I assume that the planner maximizes the sum of all agents' expected payoffs. I take the view that there is no a priori reason to exclude appropriators' welfare from the planner's considerations. Appropriation is a form of redistribution and the planner's solution institutionalizes it in taxation and enforcement.

Linear utility. The assumption of linear utility simplifies the analysis and implies that a planner does not have an inherent desire for income redistribution. Strict concavity would intensify the planner's desire to both redistribute incomes across agents and reduce the uncertainty implied by insecure property rights, and thus reinforce the results.

The cost of hiding income. I assume that the cost of hiding income from taxation is linear: a fraction of the amount hidden is lost. This specification simplifies the analysis and precludes more general cost functions studied in the literature on costly falsification. However, the focus of my paper is not the optimal design of the tax schedule but the implications of the friction in raising funds on the security of property rights. I therefore abstract from more general descriptions of the costs of hiding income and how their properties affect the tax schedule. While the marginal cost of hiding income is constant and equal to the parameter ϕ capturing the fraction of resources lost everywhere, the main results hold for all $\phi \in [0, 1]$.

Displaying more income than one has. I assume that agents cannot pretend to have produced more than they actually have. Allowing agents to do that would add three additional incentive compatibility constraints. However, at the solution of the planner's problem as it is, no agent would ever want to do so, even if it were costless. The analysis would thus be unaffected.

Meeting probabilities. I assume that the probability of any agent meeting another agent in a particular occupation equals the measure of agents in that occupation. In particular, the probability p of any agent meeting an appropriator equals the measure of appropriators, ω^a.

While this specification simplifies the analysis, it shares qualitative features with other specifications, such as the use of a constant returns to scale matching function: all else equal, more appropriators and fewer producers increase the probability of meeting an appropriator. İmrohoroğlu et al. (2000) use a similar specification.

The probability of apprehension. I assume that the probability of apprehension equals the measure of enforcement personnel in the population. This specification is unfriendly towards the results as it makes secure property rights more costly than others. Among all concave (production) functions $\vartheta : [0,1] \rightarrow [0,1]$, with $\vartheta(0) = 0$ and $\vartheta(1) = 1$, mapping the measure of enforcement personnel relative to the population into a probability of apprehension, $\vartheta(\omega^e) = \omega^e$ gives a lower probability for all $\omega^e \in (0,1)$ than any strictly concave function ϑ. More generally, the focus of the paper is on the extent of property rights security rather than on how to achieve it in terms of a mix of, say, deterrence and punishment.

Corrupt enforcement personnel (and tax collectors). One could assume that enforcement personnel may explicitly divert the resources recovered from apprehended appropriators without changing the results. The solution to the planner's problem I analyze would still be attainable, dominate all other allocations, and leave no room for such a deviation. The possibility of corrupt enforcement personnel grabbing resources themselves is captured to the extend that appropriation activities are a stylized description of unproductive redistribution of resources. Moreover, in order to focus on a planner who is facing only the friction that income can be hidden from taxation, I do not consider corruptible tax collectors (see, for instance, Hindriks et al. (1999)). I thus abstract from an analysis of government agents' incentives and their implications altogether (see, for instance, Mookherjee and Png (1995) and Acemoglu and Verdier (1998, 2000)).

Fixed Costs. The setting can be generalized to allow for the planner to incur fixed costs $\psi > 0$ whenever she hires a positive measure of enforcement personnel. One can show that, when hiding income from taxation is never entirely costless, the results hold up as long as these fixed costs are small enough. The proofs need only minor adjustments. The logic of many intermediate results is that the initial tax schedule and allocation that I show to allow for a profitable adjustment already accommodate the fixed costs ψ, while the adjustments do not affect the occupation assignments and thus the incidence of fixed costs. Given ψ, the planner's budget equation then reads

$$(1-\theta)w^e + \psi \mathbb{I}_{\{(1-\theta)>0\}} = \int_0^1 \chi_w^p t_w f(w) dw \geq 0, \tag{10}$$

where $\mathbb{I}_{\{(1-\theta)>0\}}$ takes on the value 1 if the measure of enforcement personnel hired is strictly positive, and 0 otherwise. Her objective function, which can be derived in a similar fashion as the one in Appendix B, changes to

$$\int_0^1 \chi_w^p (w - \phi(w - \zeta_w)) f(w) dw - \psi \mathbb{I}_{\{(1-\theta)>0\}}. \tag{11}$$

Assuming there exists a small minimum positive cost $\underline{\phi} \in (0,1)$ per unit of income hidden such that $\phi \in [\underline{\phi}, 1)$, there exists a strictly positive bound $\bar{\psi} > 0$ on the fixed cost such that all results hold up for any $\psi \in [0, \bar{\psi}]$. Higher fixed costs have to be paid for by raising higher taxes from all producers. Therefore, the marginal producer has to be more productive to be able to afford those higher taxes. As a consequence, the planner hires more enforcement personnel and output and thus welfare are lower.

Endogenous effort in the enforcement activity. The lack of a choice of effort to exert when an agent is employed in enforcement seems to be less relevant than the possibility of diverting recovered resources. Given the latter option, enforcement personnel would always want to ensure a high probability of apprehending appropriators so as to create opportunities to profit from. My modeling choice thus simplifies the analysis.

Capital. For simplicity, neither appropriators nor enforcement personnel use capital. If one were to argue that one group uses capital, then one could similarly argue that so does the other. (A police car is needed to chase an offender only if the offender flees by car.) Another interpretation is that the infrastructure either group uses is already in place.

5 Conclusion

I studied a model of appropriation and endogenous enforcement of property rights. I analyzed what a planner can achieve when she is unable to verify taxable incomes. I showed that, although this friction induces a binding and effective constraint on the planner's decision, it does not prevent her from implementing perfectly secure property rights. In order to do so, the planner uses a mix of redistributive taxation and employment of potential appropriators in enforcement. Frictions in the political arena may shape the objective function in society's decision making process so as to generate imperfectly secure property rights–as well as not equally secure property rights across countries. Such frictions likely arise from strategic interactions of competing social groups with conflicting interests.

Appendices

A Tax Schedule

In this section I briefly replicate the argument given in Lacker and Weinberg (1989). A mechanism consists of a message and an action space and the tax schedule maps elements of the product of those two spaces into $[-1, 1]$. By the usual argument, for any mechanism, there is a direct mechanism that replicates it and induces truthful revelation. That is, the message space is the set of possible productivities and each worker reports her true productivity. However, the message is irrelevant. Suppose two agents with different productivities w_1 and w_2 were to display the same income $\hat{w}(w_1) = \hat{w}(w_2) = \hat{w}$. By optimality of the message being the true productivity, it must be the case that for agent w_1, $t(w_1, \hat{w}) \leq t(w_2, \hat{w})$ while for agent w_2, $t(w_1, \hat{w}) \geq t(w_2, \hat{w})$, so that $t(w_1, \hat{w}) = t(w_2, \hat{w})$. That is, the same display implies the same tax payment. Therefore, we can focus on tax schedules taking into account only the displayed income.

B The Planner's Objective Function

Using the payoff expressions, the balanced budget constraint, and the definition of the probability p, which the planner understands, the planner's objective function is given by

$$
\begin{aligned}
&\int_0^1 \left[\chi_w^p \tilde{\varphi}(\sigma; w) + (1-\chi_w^p)\chi_w^e w^e + (1-\chi_w^p)(1-\chi_w^e)\nu(\sigma; w)\right] f(w)dw \\
&= \int_0^1 \chi_w^p \tilde{\varphi}(\sigma; w) f(w)dw + (1-\theta)w^e + \nu(\sigma; w)\int_0^1 (1-\chi_w^p)(1-\chi_w^e) f(w)dw \\
&= (1-\theta p)\int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w-\zeta_w)) f(w)dw + \int_0^1 \chi_w^p t_{\zeta_w} f(w)dw \\
&\quad + p\theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w-\zeta_w)) f(w)dw \\
&= \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w-\zeta_w)) f(w)dw + \int_0^1 \chi_w^p t_{\zeta_w} f(w)dw \\
&= \int_0^1 \chi_w^p (w - \phi(w-\zeta_w)) f(w)dw.
\end{aligned}
$$

C The First Best Outcome for more General Utility Functions

In this section, I briefly discuss a generalization of Proposition 1. Let $u : [0, 1] \to \mathbb{R}$ be a strictly increasing and concave function with $u[0] = 0$ and $u[1] = 1$. Then, the payoff functions are given by

$$\tilde{\varphi}(\sigma; w) = (1 - \theta p) u[w - t_w]$$
$$\nu(\sigma; w) = \theta \int_0^1 \chi_w^p u[w - t_w] f(w) dw$$

and enforcement personnel receives a payoff $u[w^e]$ where the budget balance constraint $(1 - \theta) w^e = \int_0^1 \chi_w^p t_w f(w) dw$ has to be observed. The planner's objective function is

$$\int_0^1 \chi_w^p u[w - t_w] f(w) dw + (1 - \theta) u[w^e].$$

The incentive compatibility constraints change in the obvious way. It can be verified that the unique solution to both problem (FBP) and (FBP′) with the above alterations is $\chi_w^p = 1$ and $t_w = w - \mu$ for a.e. $w \in [0, 1]$. It follows that $\theta = 1$, $\int_0^1 t_w f(w) dw = 0$, and the objective function attains the value $u[\mu]$.

D An Example Economy

In this section, I briefly provide the model outcome for an example economy. For simplicity, I assume F is the uniform distribution on $[0, 1]$. It satisfies $F(0.1) = 0.1 < 0.9$ and the elasticity of its density is always $0 < 2$. Fix $\phi \in (0, 1)$. Given that $F(w^*) = w^* = 1 - \theta$, the equilibrium requirement $h(w^*, \theta; \phi) = 0$ can be written

$$(1 - \phi) = \frac{(1 - \theta^2)(2 - \theta)}{\theta(1 + \theta(1 - \theta))}.$$

Since the right hand side is a strictly decreasing function of θ approaching ∞ with $\theta \to 0$ and 0 with $\theta \to 1$, for each $\phi \in (0, 1)$, there is a unique θ^* that implies equality. Then, all other relevant variables follow:

$$\begin{aligned} w^* &= 1 - \theta^* \\ y^* &= \frac{1}{2}(1 - w^*)(1 + w^*) \\ \omega^{e*} &= 1 - \theta^* \\ w^{e*} &= \frac{\theta^* y^*}{(1 + \theta^*(1 - \theta^*))} \\ t_{w^*}^* &= w^* - w^{e*} \\ t_w^* &= t_{w^*}^* + \phi(w - w^*). \end{aligned}$$

For comparison, the First Best tax schedule is $t_w^{fb} = w - \frac{1}{2}$ for all $w \in [0, 1]$.

E Proofs

In this section, I collect the proofs of the results in the text. It is organized in the same way as the analysis in section 3.

E.1 The First Best Outcome

Proposition 1

Proof. Consider problem (FBP). The objective function is always less than or equal to μ and equals μ if and only if $\chi_w^p = 1$ for a.e. $w \in [0,1]$ so that $\theta = 1$ implying that $\int_0^1 t_w f(w)dw = 0$. Q.E.D.

Proposition 2

Proof. Consider problem (FBP′). First, the candidate $\chi_w^p = 1$ and $t_w = w - \mu$ for a.e. $w \in [0,1]$ is a solution to problem (FBP′). To see this, note that the objective function is always less than or equal to μ and equal to μ under the candidate solution. Also $p = 0$, $\theta = 1$, and $\int_0^1 t_w f(w)dw = 0$ under the candidate solution. It follows that producers earn μ, appropriators would earn μ, and the enforcement wage is $w^e = 0$. This outcome satisfies all incentive constraints as $\omega^e = 0$. Second, it is the unique solution. To see this, notice that any other solution (which has to attain μ since our candidate does) has to satisfy $\chi_w^p = 1$ for a.e. $w \in [0,1]$ so that $\theta = 1$ and thus $\int_0^1 t_w f(w)dw = 0$. Suppose for a contradiction that there is a positive measure of agents with $w - t_w \neq \mu$, i.e., there is a positive measure of agents with $w - t_w < \mu$. But an appropriator's payoff is $\theta \int_0^1 (w - t_w) f(w)dw = \int_0^1 w f(w)dw = \mu$ so that the incentive compatibility constraint is violated for a positive measure of agents, a contradiction. Q.E.D.

E.2 The Constrained Planner

Lemma 1

Proof. I prove this lemma in two steps. First, I show that *in any solution to problem* (PP′), *there is a positive measure of producers and the constraint set contains an allocation in which there is a cutoff $w^* = \bar{w}$ such that all agents with productivity of at least w^* produce and everybody else works in enforcement.* Consider any solution to problem (PP′) and suppose for a contradiction that $\omega^p = 0$. Then, total output (i.e., the objective), the total taxes collected, and the payoff of an appropriator are all equal to zero. There exists a $w^* \in (0,1)$ and a

$t \in (0,1)$ such that $\hat{\Omega}^p = [w^*, 1]$ and $t_w = t$ for all $w \in \hat{\Omega}^p$ satisfy

$$w^* - t = w^e = F(w^*)^{-1} t \int_{w^*}^{1} f(w)dw = t\frac{(1-F(w^*))}{F(w^*)}$$
$$> (1-F(w^*)) \left(\int_{w^*}^{1} w f(w)dw - t(1-F(w^*)) \right).$$

To see this, notice that the equalities in this expression imply that $t = F(w^*)w^*$. That is, plugging in and rewriting, for the strict inequality to hold, we need to require that $w^*(1 + F(w^*)(1-F(w^*))) - \int_{w^*}^{1} w f(w)dw > 0$. Since the limit of the left hand side with $w^* \to 1$ is $1 > 0$, the claim follows by continuity. Let the $\hat{\Omega}^e = [0, w^*)$. Then, $p = 0$, $\theta = (1-F(w^*))$, $q = (1-F(w^*))$, the producers payoff is $w - t \geq w^* - t$ for all $w \in \hat{\Omega}^p$, all agents in enforcement receive w^e. No agent wants to switch to appropriation because the payoff associated with it (the right hand side of the strict inequality) is dominated and all producers and enforcers (weakly) prefer their occupation over the other one. Finally, producers prefer to display their true income and not incur any cost of falsifying it (they cannot hide all of it). This allocation is in the constraint set and increases output and thus the objective function above zero, which establishes a contradiction. For the second part of the first step, consider the allocation just described and note that, with $F(w^*) = 0.9$, $w^*(1 + 0.9(1-0.9)) - \int_{w^*}^{1} w f(w)dw \geq w^* 1.09 - \int_{w^*}^{1} f(w)dw = w^* 1.09 - 0.1 \geq 0$ because $w^* = \bar{w} \geq 0.1$ by assumption, which completes the proof of the first step.
In the second step, I now argue that *in any solution to problem* (PP′), *there exists a* $w^* \in [0,1)$ *such that* $\Omega^p = [w^*, 1]$. Consider any solution to problem (PP′) and suppose for a contradiction that there are sets W and W' with positive measures, such that $W \subseteq \Omega^p$, $W' \cap \Omega^p = \emptyset$, and $w' > w$ for all $w \in W$ and $w' \in W'$. That is, for all $w' \in W'$ and all $w \in W$,

$$\begin{aligned}
(1-\theta p)(w' - t_{\zeta_w} - \phi(w' - \zeta_w)) &= (1-\theta p)((1-\phi)w' - t_{\zeta_w} + \phi\zeta_w) \\
&> (1-\theta p)((1-\phi)w - t_{\zeta_w} + \phi\zeta_w) \\
&= (1-\theta p)(w - t_{\zeta_w} - \phi(w - \zeta_w)) \\
&\geq \max\left\{ w^e, \theta \int_0^1 \chi_w^p (w - t_w - \phi(w - \zeta_w)) f(w)dw \right\}
\end{aligned}$$

which violates at least one constraint in problem (PP′) capturing the individual optimality of occupation assignments, a contradiction. As $\omega^p > 0$, it follows that $w^* < 1$. Q.E.D.

Proposition 3

Proof. Suppose for a contradiction that a solution σ to problem (PP′) induces a nontrivial set W of producers to hide income from taxation, $\zeta_w < w$ for all $w \in W$. Then, $\Omega^p = [w^*, 1]$, the objective function attains the value

$$\int_0^1 \chi_w^p w f(w) dw - \phi \int_W (w - \zeta_w) f(w) dw,$$

and one of three possible cases holds: the wage in enforcement w^e is greater than, equal, or less than the payoff from appropriation. In each of these cases, there exists an $\epsilon > 0$ (which I show below) such that one can define an alternative tax schedule in two steps. First, let $\tilde{t}_z = t_z - \epsilon$ for all $z \in \Omega^p$ and $\tilde{t}_z = t_z$ for all $z \notin \Omega^p$. Guessing that Ω^p will remain unchanged, this change in the tax schedule does not alter any producer's optimal income display as it just adds a constant to the objective function on the constraint set in problem (1),

$$\zeta(w;\sigma) \in \arg \max_{z \in [0,w] \cap Z(\Omega^p)} \; w - \tilde{t}_z - \phi(w-z) = \arg \max_{z \in [0,w] \cap Z(\Omega^p)} \; w - t_z - \phi(w-z) + \epsilon.$$

(The planner understands that a producer displaying any $z < w^*$ is falsifying and punishes her.) All agents not in $\Omega^p = [w^*, 1]$ cannot produce and then pretend to have produced more than they have so as to be subject to the lower tax. Leaving all other elements of σ untouched, let $\tilde{\sigma}$ be the regime σ with the tax schedule t replaced by the schedule $\tilde{t}$. Second, define $\hat{t}$ by

$$\hat{t}_z = \begin{cases} \tilde{t}_{\zeta(z;\tilde{\sigma})} + \phi(z - \zeta(z;\tilde{\sigma})) = t_{\zeta(z;\sigma)} + \phi(z - \zeta(z;\sigma)) - \epsilon & \forall z \in W, \\ \tilde{t}_z & \forall z \notin W. \end{cases}$$

That is, for all producers that display their true income, the tax payment simply decreases by ϵ. At the same time, for all producers that optimally hide income from taxation, the new tax schedule implements the same after tax income plus ϵ, except with a higher tax payment and no costs incurred for falsifying income. As $\tilde{t}$ implements the same optimal income displays as t, which implies that all producers $w \notin W$ display their true income, and $\hat{t}$ offers the same maximal after-tax income without hiding the income from taxation, $\hat{t}$ implements truthful income displays for almost everywhere $w \in \Omega^p$. Still guessing that Ω^p remains unchanged, letting the regime $\hat{\sigma}$ be the regime σ with the tax schedule t replaced by $\hat{t}$, at the regime $\hat{\sigma}$ the objective function attains the value

$$\int_0^1 \chi_w^p w f(w) dw > \int_0^1 \chi_w^p w f(w) dw - \phi \int_W (w - \zeta_w) f(w) dw,$$

which contradicts σ being as solution to problem (PP′). I need to show that such an $\epsilon > 0$ exists so that $\hat{\sigma}$ is in the constraint set: the occupation instructions are individually optimal and consistent and the budget balance constraint is satisfied. Consider each case in turn.

First, suppose that the wage in enforcement is greater than the payoff from appropriation,

$$w^e > \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw.$$

Then, there are no appropriators, $p = \omega^a = 0$, and for every $\epsilon > 0$ we have that

$$w^e + \epsilon > \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw + \theta \epsilon \omega^p.$$

Budget balance then requires that, given a new wage $\hat{w}^e = w^e + \epsilon$ in enforcement, $\hat{t}$ satisfies

$$\begin{aligned}(1-\theta) w^e + (1-\theta)\epsilon = \int_0^1 \chi_w^p t_{\zeta(w;\sigma)} f(w) dw + \phi \int_W (w - \zeta(w;\sigma)) f(w) dw - \epsilon \omega^p \\ \Leftrightarrow \quad \epsilon = [(1-\theta) + \omega^p]\epsilon = \phi \int_W (w - \zeta(w;\sigma)) f(w) dw,\end{aligned}$$

because $\omega^p + \omega^e + \omega^a = 1$ and $\omega^a = 0$ and $\omega^e = (1-\theta)$. By construction, ϵ satisfies budget balance and the payoffs from production and enforcement increase by exactly ϵ for all agents,

$$(w - t_{\zeta_w} - \phi(w - \zeta_w)) + \epsilon,$$

so that no producer and no enforcer wants to switch between those two occupations–enforcers have productivity $w < w^*$, cannot pretend to have produced more than that and thus face the same tax payment as under σ, and so do not profit from switching to production. The payoff from appropriation increases by less than ϵ so that still no agent engages in appropriation. That is, all occupation assignments provided with the initial regime σ are still incentive compatible. Therefore, the regime $\hat{\sigma}$, which only replaces the tax schedule t in the regime σ is in the constraint set and attains a higher value for the objective function. Second, suppose that the wage in enforcement is less than the payoff from appropriation,

$$w^e < \theta \int_0^1 \chi_w^p (w - t_{\zeta_w} - \phi(w - \zeta_w)) f(w) dw.$$

In this case, no enforcement personnel is employed, $\theta = 1$, and all agents with productivity $w < w^*$ are appropriators. The tax schedule t is redistributive only so that

$$\int_0^1 \chi_w^p t_{\zeta(w;\sigma)} f(w) dw = 0.$$

Without loss of generality, assume that $w^e = 0$. Otherwise, simply set the wage in enforcement in the new regime to $\hat{w}^e = 0$. Let ϵ satisfy

$$\epsilon \omega^p = \phi \int_W (w - \zeta(w;\sigma)) f(w) dw.$$

Then, the budget balance constraint is satisfied, all agents prefer appropriation over working in enforcement, and for all producers $w \in \Omega^p$, initially preferring production over appropriation,

$$\begin{aligned}(1-p)(w-t_{\zeta_w}-\phi(w-\zeta_w))+(1-p)\epsilon &= (1-p)(w-t_{\zeta_w}-\phi(w-\zeta_w))+\omega^p\epsilon \\ &\geq \int_0^1 \chi_w^p(w-t_{\zeta_w}-\phi(w-\zeta_w))f(w)dw+\omega^p\epsilon,\end{aligned}$$

because $1-p=1-\omega^a=\omega^p$ as $\omega^e=0$. So, all producers under t are producers under $\hat{t}$ and all appropriators remain appropriators as they have productivity $w<w^*$, cannot pretend to have produced more than that and thus face the same tax payment as under σ, and so do not profit from switching to production. Third, suppose that the wage in enforcement equals the payoff from appropriation,

$$w^e=\theta\int_0^1 \chi_w^p(w-t_{\zeta_w}-\phi(w-\zeta_w))f(w)dw.$$

If there were no appropriators, then $p=\omega^a=0$ as in the first case and the ϵ described above for this case works fine as, due to $1\geq\theta\omega^p$, the enforcement wage increases at least as much as the payoff from appropriation. If there were no enforcement personnel, then $\theta=1$ as in the second case and the approach and ϵ described above works fine. Thus, the only case left is the one in which there are both appropriators and enforcers so that $p=\omega^a>0$ and $\theta<1$. For this last case, let $\epsilon>0$ satisfy

$$\begin{aligned}(1-\theta)w^e+(1-\theta)\theta\omega^p\epsilon &= \int_0^1 \chi_w^p t_{\zeta(w;\sigma)}f(w)dw+\phi\int_W(w-\zeta(w;\sigma))f(w)dw-\epsilon\omega^p \\ \Leftrightarrow\quad \epsilon((1-\theta)\theta+1)\omega^p &= (1-\theta)\theta\omega^p\epsilon+\epsilon\omega^p=\phi\int_W(w-\zeta(w;\sigma))f(w)dw.\end{aligned}$$

Then the tax schedule $\hat{t}$ satisfies the budget balance constraint when the wage in enforcement is $\hat{w}=w^e+\theta\omega^p\epsilon$. At the same time, the payoff from appropriation is

$$\theta\int_0^1 \chi_w^p(w-t_{\zeta_w}-\phi(w-\zeta_w))f(w)dw+\theta\epsilon\omega^p=\hat{w}^e,$$

so that no appropriator or enforcer has an incentive to switch between these two occupation while for all producers $w\in\Omega^p$, the payoff from production under $\hat{t}$ is

$$(1-\theta p)(w-t_{\zeta_w}-\phi(w-\zeta_w))+(1-\theta p)\epsilon\geq(1-\theta p)(w-t_{\zeta_w}-\phi(w-\zeta_w))+\theta\omega^p\epsilon,$$

as $1\geq\theta(\omega^p+\omega^a)$ holds, so that all producers' payoffs increase enough for them not to want to switch to enforcement or appropriation. Both appropriators and enforcers have productivity $w<w^*$, cannot pretend to have produced more than that and thus face the same tax payment as under σ, and so do not profit from switching to production. Thus, in all cases, such an ϵ

and a regime $\hat{\sigma}$ exists, which completes the proof. Q.E.D.

Lemma 2

Proof. Consider any solution to problem (PP″) and suppose for a contradiction that $\omega^e = 0$. Then, $\theta = 1$, and since, by Lemma 1, $\Omega^p = [w^*, 1]$ for some $w^* \in [0, 1)$, it has to be the case that $p = F(w^*) < 1$ and

$$(1 - F(w^*))(w^* - t_{w^*}) \geq \int_{w^*}^{1} (w - t_w) f(w) dw \;\Leftrightarrow\; (w^* - t_{w^*}) \geq \int_{w^*}^{1} (w - t_w) \frac{f(w)}{(1 - F(w^*))} dw.$$

However, that is impossible for all $w^* < 1$ because constraint (p1) implies that $w - t_w \geq (1-\phi)w + \phi w^* - t_{w^*} > w^* - t_{w^*}$ for all $w > w^*$, establishing a contradiction. Q.E.D.

Proposition 4

I prove Proposition 4 by a sequence of lemmas.

Lemma 3. *In any solution to problem (PP″), constraint (p1) holds with equality for w^*.*

Proof. Consider any solution to problem (PP″). Observe that, due to constraint (p1), $(1 - \theta p)(w^* - t_{w^*}) \geq \max\left\{w^e, \theta \int_{w^*}^{1} (w - t_w) f(w) dw\right\}$ since $\Omega^p = [w^*, 1]$ and $w^* < 1$. Suppose for a contradiction that the inequality is strict. Since $\omega^e > 0$, $w^e = \max\left\{w^e, \theta \int_{w^*}^{1} (w - t_w) f(w) dw\right\}$ and $(1 - \theta p)(w^* - t_{w^*}) > w^e = (1 - \theta)^{-1} \int_{w^*}^{1} t_w f(w) dw$. Suppose $\omega^a > 0$. Then $p = (F(w^*) - (1 - \theta))$ and we have

$$(1 - \theta(F(w^*) - (1 - \theta)))(w^* - t_{w^*}) > (1 - \theta)^{-1} \int_{w^*}^{1} t_w f(w) dw \geq \theta \int_{w^*}^{1} (w - t_w) f(w) dw.$$

There exists an $\epsilon > 0$ such that the tax schedule $\tilde{t}_w = t_w + \epsilon$ for all $w \in \Omega^p$ satisfies

$$(1 - \theta(F(w^*) - (1 - \theta)))(w^* - \tilde{t}_{w^*}) > (1 - \theta)^{-1} \int_{w^*}^{1} \tilde{t}_w f(w) dw > \theta \int_{w^*}^{1} (w - \tilde{t}_w) f(w) dw.$$

There now exists a $\delta > 0$, such that $F(w^* - \delta) - (1 - \theta - \delta) > 0$ and

$$\begin{aligned}
&(1 - (\theta + \delta)(F(w^* - \delta) - (1 - \theta - \delta)))(w^* - \delta - \tilde{t}_{w^*}) \\
&> (1 - \theta - \delta)^{-1} \left(\int_{w^*}^{1} \tilde{t}_w f(w) dw + \tilde{t}_{w^*} \int_{w^* - \delta}^{w^*} f(w) dw \right) \\
&> (\theta + \delta) \left(\int_{w^*}^{1} (w - \tilde{t}_w) f(w) dw + \int_{w^* - \delta}^{w^*} (w - \tilde{t}_{w^*}) f(w) dw \right).
\end{aligned}$$

so that $\tilde{t}_w = \tilde{t}_{w^*}$ for all agents $w \in [w^* - \delta, w^*]$. Let $\hat{w}^* = w^* - \delta$, $\hat{\Omega}^p = [\hat{w}^*, 1]$, $\hat{\theta} = \theta + \delta$,

$\hat{\omega}^e = (1-\hat{\theta})$, $\hat{p} = F(\hat{w}^*) - (1-\hat{\theta})$. Then we have

$$
\begin{aligned}
(1-\hat{\theta}\hat{p})(\hat{w}^* - \tilde{t}_{w^*}) &> (1-\hat{\theta})^{-1}\left(\int_{w^*}^{1} \tilde{t}_w f(w)dw + \tilde{t}_{w^*}\int_{\hat{w}^*}^{w^*} f(w)dw\right) \\
&> \hat{\theta}\left(\int_{w^*}^{1}(w-\tilde{t}_w)f(w)dw + \int_{\hat{w}^*}^{w^*}(w-\tilde{t}_{w^*})f(w)dw\right).
\end{aligned}
$$

Finally, there now exists a $\gamma > 0$ such that $\hat{t}_w = \tilde{t}_w - \gamma$ for all $w \in \hat{\Omega}^p$ satisfies

$$
(1-\hat{\theta}\hat{p})(\hat{w}^* - \hat{t}_{w^*}) > (1-\hat{\theta})^{-1}\int_{\hat{w}^*}^{1} \hat{t}_w f(w)dw = \hat{\theta}\int_{\hat{w}^*}^{1}(w-\hat{t}_w)f(w)dw.
$$

Letting $\hat{w}^e = (1-\hat{\theta})^{-1}\int_{\hat{w}^*}^{1}\hat{t}_w f(w)dw$, the candidate summarized by $\hat{\theta}$, $\hat{t}$, $\hat{w}^e$, $\hat{\Omega}^p$, and $\hat{\Omega}^e$ satisfies all constraints in problem (PP″) but $\hat{w}^* < w^*$ implies a higher value of the objective function, which establishes a contradiction. Now, suppose that $\omega^a = 0$. Then, $\Omega^e = [0, w^*)$, $\omega^e = (1-\theta) = F(w^*)$, so that $p = 0$, $\theta = (1-F(w^*))$, and

$$
\begin{aligned}
(1-\theta p)(w^* - t_{w^*}) &= (w^* - t_{w^*}) \\
&> F(w^*)^{-1}\int_{w^*}^{1} t_w f(w)dw \geq (1-F(w^*))\int_{w^*}^{1}(w-t_w)f(w)dw.
\end{aligned}
$$

There exists an $\epsilon > 0$ such that the tax schedule $\tilde{t}_w = t_w + \epsilon$.

$$
(w^* - \tilde{t}_w) > F(w^*)^{-1}\int_{w^*}^{1} \tilde{t}_w f(w)dw > (1-F(w^*))\int_{w^*}^{1}(w-\tilde{t}_w)f(w)dw.
$$

Then, there is a $\delta > 0$ such that

$$
\begin{aligned}
(w^* - \delta - \tilde{t}_{w^*}) &> F(w^*-\delta)^{-1}\left(\int_{w^*}^{1}\tilde{t}_w f(w)dw + \tilde{t}_{w^*}\int_{w^*-\delta}^{w^*} f(w)dw\right) \\
&> (1-F(w^*-\delta))\left(\int_{w^*}^{1}(w-\tilde{t}_w)f(w)dw + \int_{w^*-\delta}^{w^*}(w-\tilde{t}_{w^*})f(w)dw\right).
\end{aligned}
$$

Let $\hat{w}^* = w^* - \delta$, $\hat{\Omega}^p = [\hat{w}^*, 1]$, $\hat{\Omega}^e = [0, \hat{w}^*)$, $\hat{t}_w = \tilde{t}_w$ for all $w \in \hat{\Omega}^p$, $w \geq w^*$, and $\hat{t}_w = \tilde{t}_{w^*}$ for all $w \in [\hat{w}^*, w^*)$. Then we have that

$$
(\hat{w}^* - \hat{t}_{\hat{w}^*}) > F(\hat{w}^*)^{-1}\int_{\hat{w}^*}^{1}\hat{t}_w f(w)dw > (1-F(\hat{w}^*))\int_{\hat{w}^*}^{1}(w-\hat{t}_w)f(w)dw.
$$

Letting $\hat{\theta} = (1-F(\hat{w}^*))$, and $\hat{w}^e = F(\hat{w}^*)^{-1}\int_{\hat{w}^*}^{1}\hat{t}_w f(w)dw$, the candidate summarized by $\hat{\theta}$, $\hat{t}$, $\hat{w}^e$, $\hat{\Omega}^p$, and $\hat{\Omega}^e$ satisfies all constraints in problem (PP″) but $\hat{w}^* < w^*$ implies a higher value of the objective function, which establishes a contradiction. Q.E.D.

Lemma 4. *In any solution to problem* (PP″), $w^e = \theta \int_{w^*}^1 (w - t_w) f(w) dw$.

Proof. Consider any solution to problem (PP″). Since $\omega^e > 0$, $w^e \geq \theta \int_{w^*}^1 (w - t_w) f(w) dw$. Suppose for a contradiction that $w^e > \theta \int_{w^*}^1 (w - t_w) f(w) dw$. Then, $\Omega^e = [0, w^*)$, $\omega^e = (1 - \theta) = F(w^*)$, so that $p = 0$, $\theta = (1 - F(w^*))$, and, by Lemma 3, $(1 - \theta p)(w^* - t_{w^*}) = (w^* - t_{w^*}) = w^e = F(w^*)^{-1} \int_{w^*}^1 t_w f(w) dw$. There exists an $\epsilon > 0$ such that

$$(w^* - t_{w^*} + \epsilon) > F(w^*)^{-1} \int_{w^*}^1 (t_w - \epsilon) f(w) dw > (1 - F(w^*)) \int_{w^*}^1 (w - t_w + \epsilon) f(w) dw.$$

Define $\tilde{t}_w = t_w - \epsilon$. Then, there is a $\delta > 0$ such that

$$\begin{aligned}(w^* - \delta - \tilde{t}_{w^*}) &> F(w^* - \delta)^{-1} \left(\int_{w^*}^1 \tilde{t}_w f(w) dw + \tilde{t}_{w^*} \int_{w^* - \delta}^{w^*} f(w) dw \right) \\ &> (1 - F(w^* - \delta)) \left(\int_{w^*}^1 (w - \tilde{t}_w) f(w) dw + \int_{w^* - \delta}^{w^*} (w - \tilde{t}_{w^*}) f(w) dw \right).\end{aligned}$$

Let $\hat{w}^* = w^* - \delta$, $\hat{\Omega}^p = [\hat{w}^*, 1]$, $\hat{\Omega}^e = [0, \hat{w}^*)$, $\bar{t}_w = \tilde{t}_w$ for all $w \in \hat{\Omega}^p$, $w \geq w^*$, and $\bar{t}_w = \tilde{t}_{w^*}$ for all $w \in [\hat{w}^*, w^*)$. Then we have that

$$(\hat{w}^* - \bar{t}_{\hat{w}^*}) > F(\hat{w}^*)^{-1} \int_{\hat{w}^*}^1 \bar{t}_w f(w) dw > (1 - F(\hat{w}^*)) \int_{\hat{w}^*}^1 (w - \bar{t}_w) f(w) dw.$$

Now, there is a $\gamma > 0$ such that

$$(\hat{w}^* - \bar{t}_{\hat{w}^*} - \gamma) = F(\hat{w}^*)^{-1} \int_{\hat{w}^*}^1 (\bar{t}_w + \gamma) f(w) dw > (1 - F(\hat{w}^*)) \int_{\hat{w}^*}^1 (w - \bar{t}_w - \gamma) f(w) dw.$$

Letting $\hat{\theta} = (1 - F(\hat{w}^*))$, $\hat{t}_w = \bar{t}_w + \gamma$ for all $w \in \hat{\Omega}^p$, and $\hat{w}^e = F(\hat{w}^*)^{-1} \int_{\hat{w}^*}^1 \hat{t}_w f(w) dw$, the candidate summarized by $\hat{\theta}$, $\hat{t}$, $\hat{w}^e$, $\hat{\Omega}^p$, and $\hat{\Omega}^e$ satisfies all constraints in problem (PP″) but $\hat{w}^* < w^*$ implies a higher value of the objective function, which establishes a contradiction. Q.E.D.

Lemma 5. *In any solution to problem* (PP″), $\phi w - t_w = \phi w^* - t_{w^*}$ *for all* $w \in \Omega^p$.

Proof. Consider any solution to problem (PP″) and suppose for a contradiction that there is a set $W \subseteq \Omega^p$ with positive measure such that $\phi w - t_w > \phi w^* - t_{w^*}$. Notice that there exists an $\epsilon_1 > 0$ such that

$$\int_{w^*}^1 (t_{w^*} + \phi(w - w^*) - \epsilon_1) f(w) dw = \int_{w^*}^1 t_w f(w) dw.$$

Let $\tilde{t}_w = t_{w^*} + \phi(w - w^*) - \epsilon_1$ for all $w \in \Omega^p$ to get

$$(1 - \theta p)(w^* - \tilde{t}_{w^*}) > w^e = (1-\theta)^{-1} \int_{w^*}^{1} \tilde{t}_w f(w) dw \tag{12}$$
$$= (1-\theta)^{-1} \int_{w^*}^{1} t_w f(w) dw$$
$$= \theta \int_{w^*}^{1} (w - t_w) f(w) dw$$
$$= \theta \int_{w^*}^{1} (w - \tilde{t}_w) f(w) dw,$$

where the second to last equality is due to Lemma 4 and $p = F(w^*) - (1-\theta)$. Suppose that $\omega^a > 0$. There exists an $\epsilon_2 > 0$ such that the tax schedule $\bar{t}_w = \tilde{t}_w + \epsilon_2$ for all $w \in \Omega^p$ satisfies

$$(1 - \theta(F(w^*) - (1-\theta)))(w^* - \bar{t}_{w^*}) > (1-\theta)^{-1} \int_{w^*}^{1} \bar{t}_w f(w) dw > \theta \int_{w^*}^{1} (w - \bar{t}_w) f(w) dw.$$

There now exists an $\delta > 0$, such that $F(w^* - \delta) - (1 - \theta - \delta) > 0$ and

$$\begin{aligned} &(1 - (\theta+\delta)(F(w^* - \delta) - (1 - \theta - \delta)))(w^* - \delta - \bar{t}_{w^*}) \\ &> (1 - \theta - \delta)^{-1} \left(\int_{w^*}^{1} \bar{t}_w f(w) dw + \bar{t}_{w^*} \int_{w^* - \delta}^{w^*} f(w) dw \right) \\ &> (\theta + \delta) \left(\int_{w^*}^{1} (w - \bar{t}_w) f(w) dw + \int_{w^* - \delta}^{w^*} (w - \bar{t}_{w^*}) f(w) dw \right). \end{aligned}$$

so that $\bar{t}_w = \bar{t}_{w^*}$ for all agents $w \in [w^* - \delta, w^*]$. Let $\hat{w}^* = w^* - \delta$, $\hat{\Omega}^p = [\hat{w}^*, 1]$, $\hat{\theta} = \theta + \delta$, $\hat{\omega}^e = (1 - \hat{\theta})$, $\hat{p} = F(\hat{w}^*) - (1 - \hat{\theta})$. Then we have

$$\begin{aligned} (1 - \hat{\theta}\hat{p})(\hat{w}^* - \bar{t}_{w^*}) &> (1 - \hat{\theta})^{-1} \left(\int_{w^*}^{1} \bar{t}_w f(w) dw + \bar{t}_{w^*} \int_{\hat{w}^*}^{w^*} f(w) dw \right) \\ &> \hat{\theta} \left(\int_{w^*}^{1} (w - \bar{t}_w) f(w) dw + \int_{\hat{w}^*}^{w^*} (w - \bar{t}_{w^*}) f(w) dw \right). \end{aligned}$$

There now exists an $\epsilon_3 > 0$ such that $\hat{t}_w = \bar{t}_w - \epsilon_3$ for all $w \in \hat{\Omega}^p$ satisfies

$$(1 - \hat{\theta}\hat{p})(\hat{w}^* - \hat{t}_{w^*}) > (1 - \hat{\theta})^{-1} \int_{\hat{w}^*}^{1} \hat{t}_w f(w) dw = \hat{\theta} \int_{\hat{w}^*}^{1} (w - \hat{t}_{w^*}) f(w) dw.$$

Letting $\hat{w}^e = (1 - \hat{\theta})^{-1} \int_{\hat{w}^*}^{1} \hat{t}_w f(w) dw$, the candidate summarized by $\hat{\theta}$, $\hat{t}$, $\hat{w}^e$, $\hat{\Omega}^p$, and $\hat{\Omega}^e$ satisfies all constraints in problem (PP$''$) but $\hat{w}^* < w^*$ implies a higher value of the objective function, which establishes a contradiction. Now, suppose that $\omega^a = 0$. Then, $\Omega^e = [0, w^*)$,

$\omega^e = (1-\theta) = F(w^*)$, so that $p = 0$, $\theta = (1 - F(w^*))$, and equation (12) becomes

$$
\begin{aligned}
(1-\theta p)(w^* - \tilde{t}_{w^*}) &= (w^* - \tilde{t}_{w^*}) \\
&> F(w^*)^{-1} \int_{w^*}^{1} \tilde{t}_w f(w)dw = (1 - F(w^*)) \int_{w^*}^{1} (w - \tilde{t}_w) f(w)dw.
\end{aligned}
$$

There exists an $\epsilon_2 > 0$ such that the tax schedule $\bar{t}_w = \tilde{t}_w + \epsilon_2$ for all $w \in \Omega^p$ satisfies

$$
(w^* - \bar{t}_w) > F(w^*)^{-1} \int_{w^*}^{1} \bar{t}_w f(w)dw > (1 - F(w^*)) \int_{w^*}^{1} (w - \bar{t}_w) f(w)dw.
$$

Then, there is a $\delta > 0$ such that

$$
\begin{aligned}
(w^* - \delta - \bar{t}_{w^*}) &> F(w^* - \delta)^{-1} \left(\int_{w^*}^{1} \bar{t}_w f(w)dw + \bar{t}_{w^*} \int_{w^*-\delta}^{w^*} f(w)dw \right) \\
&> (1 - F(w^* - \delta)) \left(\int_{w^*}^{1} (w - \bar{t}_w) f(w)dw + \int_{w^*-\delta}^{w^*} (w - \bar{t}_{w^*}) f(w)dw \right).
\end{aligned}
$$

Let $\hat{w}^* = w^* - \delta$, $\hat{\Omega}^p = [\hat{w}^*, 1]$, $\hat{\Omega}^e = [0, \hat{w}^*)$, $\hat{t}_w = \bar{t}_w$ for all $w \in \hat{\Omega}^p$, $w \geq w^*$, and $\hat{t}_w = \bar{t}_{w^*}$ for all $w \in [\hat{w}^*, w^*)$. Then we have that

$$
(\hat{w}^* - \hat{t}_{\hat{w}^*}) > F(\hat{w}^*)^{-1} \int_{\hat{w}^*}^{1} \hat{t}_w f(w)dw > (1 - F(\hat{w}^*)) \int_{\hat{w}^*}^{1} (w - \hat{t}_w) f(w)dw.
$$

Letting $\hat{\theta} = (1 - F(\hat{w}^*))$ and $\hat{w}^e = F(\hat{w}^*)^{-1} \int_{\hat{w}^*}^{1} \hat{t}_w f(w)dw$, the candidate summarized by $\hat{\theta}$, $\hat{t}$, $\hat{w}^e$, $\hat{\Omega}^p$, and $\hat{\Omega}^e$ satisfies all constraints in problem (PP$''$) but $\hat{w}^* < w^*$ implies a higher value of the objective function, which establishes a contradiction. Q.E.D.

To summarize, the solution to problem (PP$''$) is constrained to satisfy

$$(1 - (F(w^*) - (1-\theta))\theta)(w^* - t_{w^*}) = w^e, \tag{13}$$

$$w^e = \theta \int_{w^*}^{1} (w - t_w) f(w)dw, \tag{14}$$

$$(1-\theta)w^e = \int_{w^*}^{1} t_w f(w)dw, \tag{15}$$

$$\phi w - t_w = \phi w^* - t_{w^*}, \tag{16}$$

$$F(w^*) \geq (1-\theta). \tag{17}$$

I can now prove Proposition 4.

Proof. Consider problem (PP$''$) on the constraint set (13)-(17). I first show that one can summarize the constraint set as one constraint on w^*: given w^*, there exist unique θ, w^e, and tax schedule t that satisfy all the constraints. Then, I find the optimal w^* and derive the allocation. Combining equations (14) and (15), since $\theta < 1$, the tax receipts collected are

$$\int_{w^*}^{1} t_w f(w)dw = \frac{\theta(1-\theta)}{1+\theta(1-\theta)} \int_{w^*}^{1} wf(w)dw, \tag{18}$$

so that

$$w^e = \frac{\theta}{1+\theta(1-\theta)} \int_{w^*}^{1} wf(w)dw. \tag{19}$$

Using this expression in equation (13) gives

$$(1-(F(w^*)-(1-\theta))\theta)(w^* - t_{w^*}) = \frac{\theta}{1+\theta(1-\theta)} \int_{w^*}^{1} wf(w)dw. \tag{20}$$

Now, equations (13) and (14) give

$$(1-(F(w^*)-(1-\theta))\theta)(w^* - t_{w^*}) = \theta \int_{w^*}^{1} (w - t_w)f(w)dw$$

and equation (16) implies that $w - t_w = (1-\phi)w + \phi w^* - t_{w^*}$ so that combining yields

$$\begin{aligned} &(1-(F(w^*)-(1-\theta))\theta)(w^* - t_{w^*}) \\ &= \theta(1-\phi)\left(\int_{w^*}^{1} wf(w)dw - w^*(1-F(w^*))\right) + \theta(w^* - t_{w^*})(1-F(w^*)). \end{aligned} \tag{21}$$

Solving both (20) and (21) for $(w^* - t_{w^*})$, equalizing the resulting expressions, and rewriting gives

$$(1-\phi)\left(1 - \frac{w^*(1-F(w^*))}{\int_{w^*}^{1} wf(w)dw}\right) = \frac{1-\theta^2}{(1+\theta(1-\theta))(1-(F(w^*)-(1-\theta))\theta)}. \tag{22}$$

Define the function $h : (0,1) \times [0,1] \times [0,1) \to \mathbb{R}$ given by

$$h(w^*, x; \phi) = (1-\phi)\left(1 - \frac{w^*(1-F(w^*))}{\int_{w^*}^{1} wf(w)dw}\right) - \frac{1-x^2}{(1+x(1-x))(1-(F(w^*)-(1-x))x)}. \tag{23}$$

Fix $(w^*, \phi) \in (0,1) \times [0,1)$. It follows that

$$\begin{aligned} \lim_{x\to 0} h(w^*, x; \phi) &= (1-\phi)\left(1 - \frac{w^*(1-F(w^*))}{\int_{w^*}^{1} wf(w)dw}\right) - 1 < 0, \\ \lim_{x\to 1} h(w^*, x; \phi) &= (1-\phi)\left(1 - \frac{w^*(1-F(w^*))}{\int_{w^*}^{1} wf(w)dw}\right) > 0, \end{aligned}$$

and

(24)

$$h_x(w^*, x; \phi) = \frac{2x(1 + x(1-x))(1 - x(F(w^*) - (1-x)))}{(1 + x(1-x))^2(1 - (F(w^*) - (1-x))x)^2}$$
$$+ \frac{(1-x^2)[(1-2x)(1 - (F(w^*) - (1-x))x) + (1 + x(1-x))(1 - 2x - F(w^*))]}{(1 + x(1-x))^2(1 - (F(w^*) - (1-x))x)^2} > 0$$

if and only if

$$F(w^*) < \frac{2(1 - 2x^2 + 2x^3 + x^4 - x^5)}{1 + 2x - 2x^2 + x^4}. \tag{25}$$

It can be shown that the right hand side of (25) is strictly greater than 0.9 for all $x \in [0, 1]$. By Lemma 1, $\bar{w}$ is attainable so that a solution must satisfy $w^* \leq \bar{w}$ and $F(w^*) \leq 0.9$ (because the objective function decreases in w^*) and (25) must hold. Therefore, I restrict attention to candidate solutions that satisfy $w^* \leq \bar{w}$. I then check whether the candidate solution obtained is consistent with this conjecture. If there exists at least one such a candidate solution, then the smallest candidate w^* is the unique solution to the planner's problem because the objective function decreases in w^*. Therefore, under this conjecture it follows that there exists a unique $\theta^* \in (0, 1)$ such that $h(w^*, \theta^*; \phi) = 0$. At θ^*,

$$h_\phi(w^*, \theta^*; \phi) = -\left(1 - \frac{w^*(1 - F(w^*))}{\int_{w^*}^1 wf(w)dw}\right) < 0 \quad \forall w^* \in (0, 1)$$
$$h_{w^*}(w^*, \theta^*; \phi) = -(1-\phi)\frac{(1 - F(w^*) - w^*f(w^*))\int_{w^*}^1 wf(w)dw + (w^*)^2 f(w^*)(1 - F(w^*))}{\left(\int_{w^*}^1 wf(w)dw\right)^2}$$
$$- \frac{(1 - \theta^{*2})\theta^* f(w^*)}{(1 + \theta^*(1-\theta^*))^2(1 - (F(w^*) - (1-\theta^*))\theta^*)^2} < 0$$

since, $(1 - F(w^*) - w^*f(w^*))\int_{w^*}^1 wf(w)dw + (w^*)^2 f(w^*)(1 - F(w^*))$ equals μ at $w^* = 0$ and 0 at $w^* = 1$ and has derivative

$$-(1 - F(w^*))w^*f(w^*) - [2f(w^*) + w^*f'(w^*)](1 - F(w^*))\left(\int_{w^*}^1 w\frac{f(w)}{1 - F(w^*)}dw - w^*\right) \leq 0$$

because $-\frac{w^*f'(w^*)}{f(w^*)} \leq 2$. Therefore, $(1 - F(w^*) - w^*f(w^*))\int_{w^*}^1 wf(w)dw + (w^*)^2 f(w^*)(1 - F(w^*)) \geq 0$ holds. The conditions of the implicit function theorem are satisfied. It follows that there is a function $\theta^* : (0, \bar{w}] \times [0, 1) \to [0, 1]$ with arguments (w^*, ϕ) that is differentiable on the interior of its domain with derivatives $\theta^*_{w^*}(w^*; \phi) > 0$ and $\theta^*_\phi(w^*; \phi) > 0$ such that $h(w^*, \theta^*(w^*; \phi); \phi) = 0$. That is, given $\phi \in [0, 1)$, the constraint set can be summarized by the function $\theta^*(w^*; \phi)$ and $F(w^*) \geq (1 - \theta^*(w^*; \phi))$ and the planner just chooses $w^* \in (0, 1)$, which then has to be checked for $F(w^*) \leq 0.9$. Since, given ϕ, the objective function is strictly decreasing in w^* and $\theta^*_{w^*}(w^*;\phi) > 0$, the solution minimizes w^* subject to the constraint $F(w^*) \geq (1-\theta^*(w^*;\phi))$. Let $g : (0,\bar{w}] \times [0,1) \to [0,1]$ be the differentiable function given by $g(w^*;\phi) = F(w^*) - 1 + \theta^*(w^*;\phi)$. The constraint can then be written as $g(w^*;\phi) \geq 0$. Now, $\lim_{w^*\to 0} g(w^*;\phi) = \lim_{w^*\to 0} \theta^*(w^*;\phi) - 1 < 0$ and $\lim_{w^*\to\bar{w}} g(w^*;\phi) = \theta^*(\bar{w};\phi) - 0.1 > 0$, because $\theta^*(\bar{w},0) \leq \theta^*(\bar{w},\phi)$ for all $\phi \in [0,1)$,

$$\begin{aligned}
h(\bar{w},0.1;0) &= 1 - \frac{\bar{w}(1-F(\bar{w}))}{\int_{\bar{w}}^1 wf(w)dw} - \frac{1-0.1^2}{(1+0.1(1-0.1))(1-(F(\bar{w})-(1-0.1))0.1)} \\
&= 1 - \frac{\bar{w}0.1}{\int_{\bar{w}}^1 wf(w)dw} - \frac{0.99}{1.09} < 0.0917 - \frac{0.1^2}{\int_{\bar{w}}^1 f(w)dw} = 0.0917 - 0.1 < 0,
\end{aligned}$$

and $h_x(w^*,x;\phi) > 0$ so that $\theta^*(\bar{w},\phi) \geq \theta^*(\bar{w},0) > 0.1$ for all $\phi \in [0,1)$. It therefore follows from $g_{w^*}(w^*;\phi) = f(w^*) + \theta^*_{w^*}(w^*;\phi) > 0$ that the unique solution to the planner's problem implies that the constraint is binding. That is, given $\phi \in [0,1)$, $w^*(\phi)$ solves $g(w^*(\phi);\phi) = 0$ or $F(w^*(\phi)) = (1-\theta^*(w^*(\phi);\phi))$; $\theta^*(w^*(\phi);\phi)$ follows. This solution is consistent with expression (25) independent of assumptions on F. Imposing $F(w^*) = 1-x$, it can be rewritten as $(1-x)(1+x^4) + 2x^3 > 0$, which holds for all $x \in [0,1]$. Again by the implicit function theorem, there is a differentiable function $w^* : [0,1) \to (0,1)$, such that $g(w^*(\phi);\phi) = 0$ with derivative $w^{*\prime}(\phi) = -\frac{g_\phi(w^*;\phi)}{g_{w^*}(w^*;\phi)} = \frac{-\theta^*_\phi(w^*;\phi)}{f(w^*)+\theta^*_{w^*}(w^*;\phi)} < 0$. Since $\omega^e = (1-\theta) = F(w^*)$, it follows that $\omega^a = 0$ and property rights are perfectly secure. The tax schedule is given by $t_w = (t_{w^*} - \phi w^*) + \phi w$, which is a linear and strictly increasing function of w if $\phi > 0$ and constant in w if $\phi = 0$; w^e follows from equation (19).

Consider an increase in ϕ. Since $w^{*\prime}(\phi) < 0$, output and welfare increase. Also, $F(w^*)$ decreases and from $F(w^*) = (1-\theta^*)$ it follows that θ^* increases and $\omega^e = F(w^*)$ decreases. Since $w^{e*} = \frac{\theta^*}{1+\theta^*(1-\theta^*)}\int_{w^*}^1 wf(w)dw = \frac{1}{(1-F(w^*))^{-1}+F(w^*)}\int_{w^*}^1 wf(w)dw$ and w^* decreases, w^{e*} increases. Since at the solution $w^* - t_{w^*} = w^{e*}$, $t_{w^*} = w^* - w^{e*}$ then decreases. The tax schedule $t^*_w = t^*_{w^*} + \phi(w-w^*)$ is steeper since ϕ increased.

Finally, since $w^{*\prime}(\phi) < 0$, $w^*(0) \geq w^*(\phi)$ for all $\phi \in [0,1)$. Under the maintained technical assumptions, replacing $(1-\theta) = F(w^*)$ and $\theta = 1-F(w^*)$ in equation (23) at $\phi = 0$ defines

$$\bar{h}(w^*) \equiv h(w^*,(1-F(w^*));0) = 1 - \frac{w^*(1-F(w^*))}{\int_{w^*}^1 wf(w)dw} - \frac{F(w^*)(2-F(w^*))}{1+F(w^*)(1-F(w^*))},$$

with derivative $\bar{h}'(w^*)|_{w^*=\bar{w}} < 0$ and strictly negative $\bar{h}(\bar{w})$:

$$\begin{aligned}
\bar{h}(\bar{w}) &= 1 - \frac{\bar{w}0.1}{\int_{\bar{w}}^1 wf(w)dw} - \frac{0.99}{1.09} \\
&\leq 1 - \frac{0.1^2}{\int_{\bar{w}}^1 f(w)dw} - \frac{0.99}{1.09} = 0.9 - \frac{0.99}{1.09} = \frac{0.981-0.99}{1.09} < 0.
\end{aligned}$$

It follows that $w^*(\phi)|_{\phi=0} = w^*(0) < \bar{w}$ so that $\bar{h}(w^*(0)) = 0$. Therefore, for all $\phi \in [0,1)$, $F(w^*(\phi)) \leq F(w^*(0)) < F(\bar{w}) = 0.9$, which completes the proof. Q.E.D.

E.3 A Political Economy Friction

Proposition 5

Proof. Instead of problem (DP) consider the following problem.

$$\text{(DP}'\text{)} \qquad \max_{\sigma\in\Sigma} \int_0^1 \chi_w^p t_w f(w)dw - (1-\theta)w^e$$

$$\text{(26)} \quad s.t. \quad (1-\theta p)(w-t_w) \geq (1-\theta p)\alpha \quad \text{for a.e. } w \in \Omega^p,$$

$$\text{(27)} \qquad w^e \geq \theta \int_0^1 \chi_w^p (w-t_w) f(w)dw \quad \text{for a.e. } w \in \Omega^e,$$

$$(1-\theta)w^e \geq 0,\ \omega^p + \omega^e + \omega^a = 1,\ \omega^p, \omega^e, \omega^a \geq 0,\ q = \omega^p,\ (1-\theta) = \omega^e,\ p = \omega^a.$$

I characterize the solution of problem (DP′) and then show that it is attainable in problem (DP). Since the constraint set in problem (DP) is contained in the constraint set of problem (DP′), the solution of problem (DP′) is the solution of problem (DP). Consider any solution to problem (DP′). I first analyze an interior candidate solution and then show that it dominates the relevant boundary candidate solution. Suppose that $\omega^p > 0$ so that $p = \omega^a < 1$ and thus $(1-\theta p) > 0$ for all $\theta \in [0,1]$. Then, constraint (26) is satisfied if and only if $t_w \leq w - \alpha$. I proceed in steps. First, no enforcement personnel is hired so that $\omega^e = 0$ and $\theta = 1$. Suppose for a contradiction that $\omega^e = 1-\theta > 0$. By assumption, $\omega^p > 0$ so that $\omega^e = 1-\theta < 1$ implying that $\theta > 0$. Then, constraint (27) together with $w - t_w \geq \alpha$, $\omega^p > 0$, and $\theta > 0$ implies that $w^e \geq \theta \int_0^1 \chi_w^p (w-t_w) f(w)dw > 0$ so that $(1-\theta)w^e > 0$. But neither θ nor w^e affect constraint (26). It follows that simply not hiring these agents in enforcement and setting $\hat{\theta} = 1$ satisfies constraints (26) and (27) while increasing the objective since $(1-\hat{\theta})w^e = 0$, which yields a contradiction. Thus, the expenditure on enforcement is equal to zero and the rest of the proof can set $\theta = 1$, i.e., $\chi_w^e = 0$ for a.e. $w \in [0,1]$, $w^e = 0$, and ignore constraint (27) without loss of generality (w^e needs to be sufficiently small for consistency, $w^e = 0$ does the job). Second, all tax payments are non-negative, i.e., $t_w \geq 0$ for a.e. $w \in \Omega^p$. Suppose there is a set of agents $W \subseteq \Omega^p$ with positive measure such that $t_w < 0$ for all $w \in W$. Replace the original occupational assignments by $\hat{\chi}_w^p = \chi_w^p$ for all $w \notin W$ and $\hat{\chi}_w^p = 0$ for all $w \in W$. Then, $\int_0^1 \hat{\chi}_w^p t_w f(w)dw > \int_0^1 \chi_w^p t_w f(w)dw$, which yields a contradiction. Thus, $t_w \geq 0$ for a.e. $w \in \Omega^p$. Third, (26) is binding, i.e., $(1-p)(w-t_w) = (1-p)\alpha$ and $t_w = w - \alpha$ for a.e. $w \in \Omega^p$. Suppose for a contradiction that there is a set $W \subseteq \Omega^p$ with positive measure such that $t_w < w - \alpha$ for all $w \in W$. Replace the original tax schedule by the one given by $\hat{t}_w = t_w$ for all $w \in \Omega^p \backslash W$ and $\hat{t}_w = w - \alpha$ for all $w \in W$. Then, $\int_0^1 \chi_w^p \hat{t}_w f(w)dw > \int_0^1 \chi_w^p t_w f(w)dw$ and (26) is satisfied, which yields a contradiction. Thus, $t_w = w - \alpha$ for a.e. $w \in \Omega^p$. Fourth, there exists a $w^* \in (0,1)$ such that $\Omega^p = [w^*, 1]$. Suppose for a contradiction that there are sets W and W' with positive measure such that $W \subseteq \Omega^p$ and $W' \cap \Omega^p = \emptyset$ and $w' > w$ for all $w \in W$ and $w' \in W'$. Replace the original occupational assignments by $\hat{\chi}^p_w = \chi^p_w$ for all $w \notin W'$ and $\hat{\chi}^p_w = 1$ for all $w \in W'$ and set $\hat{t}_w = t_w$ for all $w \notin W'$ and $\hat{t}_w = w - \alpha$ for all $w \in W'$. Then, $\int_0^1 \hat{\chi}^p_w \hat{t}_w f(w)dw > \int_0^1 \chi^p_w t_w f(w)dw$ since $0 \leq \hat{t}_w = w - \alpha < w' - \alpha = \hat{t}_{w'}$ for all $w \in W$ and $w' \in W'$, which yields a contradiction. Thus, if $w^* \in \Omega^p$, then $w \in \Omega^p$ for all $w \geq w^*$. By assumption, $w^* < 1$ as $\omega^p > 0$. Now, suppose for a contradiction that $w^* = 0$. Then, $\Omega^p = [0,1]$ and since $t_w = w - \alpha$, $t_w < 0$ for all $w \in [0, \alpha)$, which is a set with measure $F(\alpha) > 0$, contradicting the fact that $t_w \geq 0$ for a.e. $w \in \Omega^p$. Thus, $w^* \in (0,1)$. Fifth, $t_{w^*} = 0$. Suppose for a contradiction that $t_{w^*} > 0$. Since $t_{w^*} = w^* - \alpha$, there exists an $\epsilon > 0$ such that $w^* - \epsilon - \alpha > 0$. Replace the original occupational assignments by $\hat{\chi}^p_w = \chi^p_w$ for all $w \notin [w^* - \epsilon, w^*)$ and $\hat{\chi}^p_w = 1$ for all $w \in [w^* - \epsilon, w^*)$ and set the tax schedule $\hat{t}_w = w - \alpha$ for all $w \in [w^* - \epsilon, 1]$. Then, $\int_0^1 \hat{\chi}^p_w \hat{t}_w f(w)dw > \int_0^1 \chi^p_w t_w f(w)dw$, which yields a contradiction. Thus, $t_{w^*} = 0$. It then follows directly from the tax rule that $w^* = \alpha$. The objective function takes the value $\int_\alpha^1 w f(w)dw - (1 - F(\alpha))\alpha > 0$. Suppose $\omega^p = 0$. Then, the objective is equal to zero. Therefore, the interior candidate solution is the unique solution to problem (DP′). Collecting observations gives $w^* = \alpha$, $\Omega^p = [\alpha, 1]$, $t_w = w - \alpha$, $\omega^p = (1 - F(\alpha))$, $p = \omega^a = F(\alpha) > 0$, and $\omega^e = 0$. Any producer's expected payoff is given by $(1 - F(\alpha))\alpha$, an appropriator's expected payoff is given by $\int_\alpha^1 \alpha f(w)dw = (1 - F(\alpha))\alpha$, the expected payoff in the informal sector is $(1 - F(\alpha))\alpha$, and $w^e = 0$. That is, this solution of problem (DP′) is the unique solution of problem (DP). Q.E.D.

References

Acemoglu, D. (1995). Reward Structures and the Allocation of Talent. *European Economic Review 39*(1), 17–33.

Acemoglu, D. (2003). Why Not a Political Coase Theorem? Social Conflict, Commitment and Politics. *Journal of Comparative Economics 31*(4), 620–652.

Acemoglu, D. (2006). A Simple Model of Inefficient Institutions. *The Scandinavian Journal of Economics 108*(4), 515–546.

Acemoglu, D. (2008). Oligarchic Versus Democratic Societies. *Journal of the European Economic Association 6*(1), 1–44.

Acemoglu, D. and S. Johnson (2005). Unbundling Institutions. *The Journal of Political Economy 113*(5), 949–995.

Acemoglu, D., S. Johnson, and J. A. Robinson (2005). Institutions as the Fundamental Cause of Long-Run Growth. In Handbook of Economic Growth, eds. Philippe Aghion and Stephen Durlauf, Elsevier Science.

Acemoglu, D. and J. A. Robinson (2001). Inefficient Redistribution. *The American Political Science Review 95*(3), 649–661.

Acemoglu, D. and T. Verdier (1998). Property Rights, Corruption and the Allocation of Talent: A General Equilibrium Approach. *The Economic Journal 108*, 1381–1403.

Acemoglu, D. and T. Verdier (2000). The Choice between Market Failures and Corruption. *The American Economic Review 90*, 194–211.

Allingham, M. G. and A. Sandmo (1972). Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics 1*, 323–338.

Andreoni, J., B. Erard, and J. Feinstein (1998). Tax Compliance. *Journal of Economic Literature 36*(2), 818–860.

Barro, R. J. (1990). Government Spending in a Simple Model of Endogenous Growth. *The Journal of Political Economy 98*(5), S103–S125.

Barro, R. J. (1996). Democracy and Growth. *Journal of Economic Growth 1*(1), 1–27.

Baumol, W. J. (1990). Entrepreneurship: Productive, Unproductive, and Destructive. *The Journal of Political Economy 98*(5), 893–921.

Becker, G. S. (1968). Crime and Punishment: An Economic Approach. *The Journal of Political Economy 76*(2), 169–217.

Boadway, R. and M. Keen (1993). Public Goods, Self-Selection and Optimal Income Taxation. *International Economic Review 34*(3), 463–478.

Brett, C. and J. A. Weymark (2008). Public Good Provision and the Comparative Statics of Optimal Nonlinear Income Taxation. *International Economic Review 49*(1), 255–290.

Cowell, F. A. and J. P. F. Gordon (1988). Unwillingness to Pay: Tax Evasion and Public Good Provision. *Journal of Public Economics 36*, 305–321.

Easterly, W. and R. Levine (2003). Tropics, Germs, and Crops: How Endowments Influence Economic Development. *Journal of Monetary Economics 50*(1), 3–39.

Falkinger, J. (1991). On Optimal Public Good Provision with Tax Evasion. *Journal of Public Economics 45*, 127–133.

Glomm, G. and B. Ravikumar (1994). Public Investment in Infrastructure in a Simple Growth Model. *Journal of Economic Dynamics and Control 18*, 1173–1187.

Gonzalez, F. M. and H. M. Neary (2008). Prosperity without Conflict. *Journal of Public Economics 92*, 2170–2181.

Grochulski, B. (2007). Optimal Nonlinear Income Taxation with Costly Tax Avoidance. *Federal Reserve Bank of Richmond Economic Quarterly 93*(1), 77–109.

Grossman, H. I. and M. Kim (1995). Swords or Plowshares? A Theory of the Security of Claims to Property. *The Journal of Political Economy 103*(6), 1275–1288.

Grossman, H. I. and M. Kim (1996). Predation and Accumulation. *Journal of Economic Growth 1*(3), 333–350.

Hall, R. E. and C. I. Jones (1999). Why Do Some Countries Produce So Much More Output Per Worker Than Others? *The Quarterly Journal of Economics 114*(1), 83–116.

Hindriks, J., M. Keen, and A. Muthoo (1999). Corruption, Extortion and Evasion. *Journal of Public Economics 74*, 395–430.

İmrohoroğlu, A., A. Merlo, and P. Rupert (2000). On the Political Economy of Income Redistribution and Crime. *International Economic Review 41*(1), 1–26.

Knack, S. and P. Keefer (1995). Institutions and Economic Performance: Cross-Country Tests Using Alternative Institutional Measures. *Economics and Politics 7*(3), 207–227.

Koeppl, T., C. Monnet, and E. Quintin (2014). Efficient Contract Enforcement. *Economic Theory 55*, 161–183.

Lacker, J. M. and J. A. Weinberg (1989). Optimal Contracts under Costly State Falsification. *The Journal of Political Economy 97*(6), 1345–1363.

Lockwood, B. B., C. G. Nathanson, and E. G. Weyl (2015). Taxation and the Allocation of Talent. Working Paper (November 2015).

Mookherjee, D. and I. P. L. Png (1995). Corruptible law enforcers: How should they be compensated. *The Economic Journal 105*, 145–159.

Moselle, B. and B. Polak (2001). A Model of a Predatory State. *The Journal of Law, Economics, and Organization 17*(1), 1–33.

Murphy, K. M., A. Shleifer, and R. W. Vishny (1991). The Allocation of Talent: Implications for Growth. *The Quarterly Journal of Economics 106*(2), 503–530.

Murphy, K. M., A. Shleifer, and R. W. Vishny (1993). Why Is Rent-Seeking So Costly to Growth? *The American Economic Review, Papers and Proceedings 83*(2), 409–414.

Myerson, R. B. (1979). Incentive Compatibility and the Bargaining Problem. *Econometrica 47*(1), 61–74.

Reinganum, J. F. and L. L. Wilde (1985). Income Tax Compliance in a Principal-Agent Framework. *Journal of Public Economics 26*, 1–18.

Rodrik, D., A. Subramanian, and F. Trebbi (2004). Institutions Rule: The Primacy of Institutions over Geography and Integration in Economic Development. *Journal of Economic Growth 9*(2), 131–165.

Rose-Ackerman, S. (1975). The Economics of Corruption. *Journal of Public Economics 4*(2), 187–203.

Rothschild, C. and F. Scheuer (2015). Optimal Taxation with Rent-Seeking. Working Paper (October 2015).

Schneider, F. and D. H. Enste (2000). Shadow Economies: Size, Causes, and Consequences. *Journal of Economic Literature 38*(1), 77–114.

Skogh, G. and C. Stuart (1982). A Contractarian Theory of Property Rights and Crime. *The Scandinavian Journal of Economics 84*(1), 27–40.

Teng, J. (2000). Endogenous Authoritarian Property Rights. *Journal of Public Economics 77*, 81–95.

Tullock, G. (1967). The Welfare Costs of Tariffs, Monopolies, and Theft. *Western Economic Journal 5*(3), 224–232.